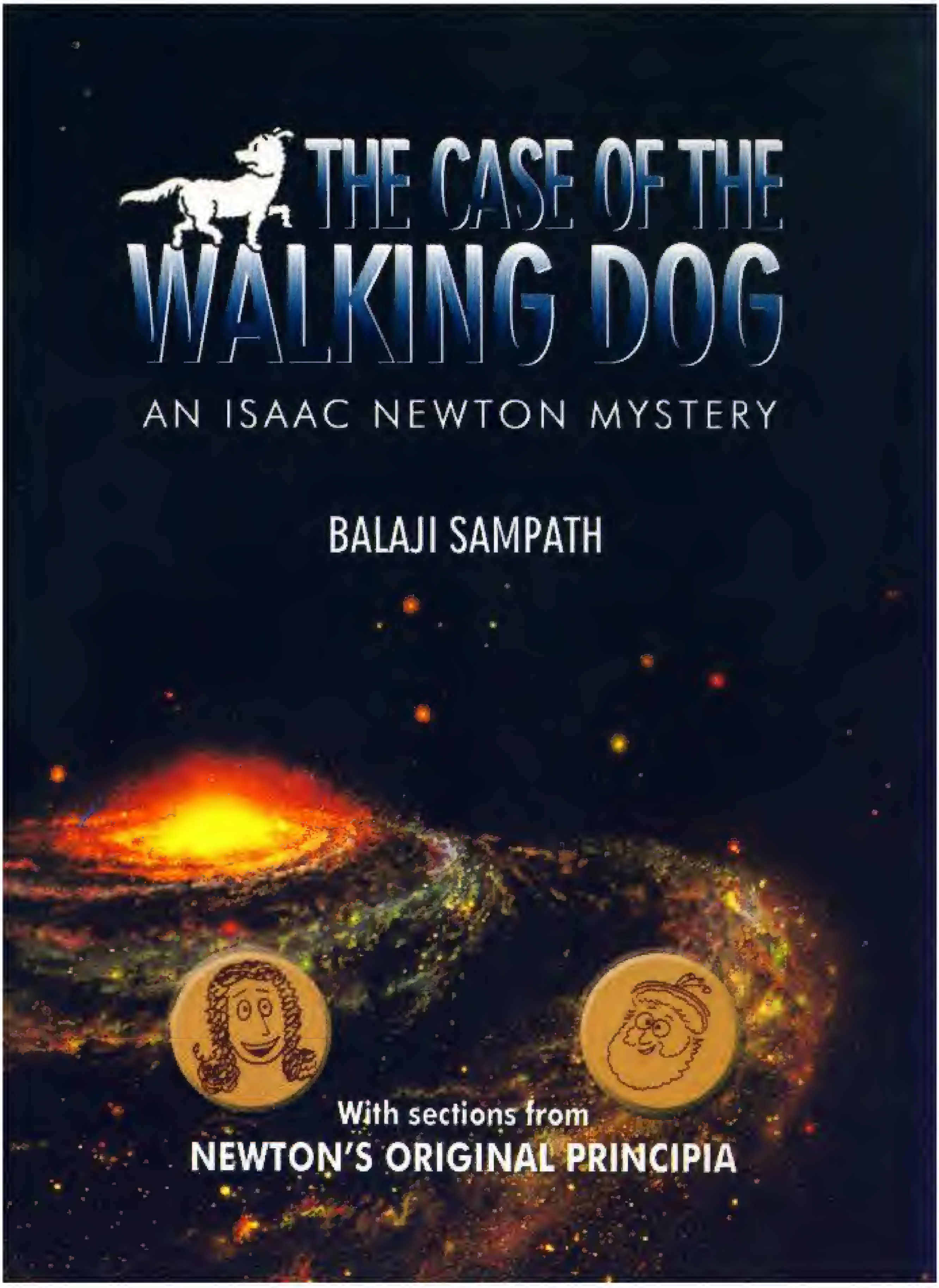
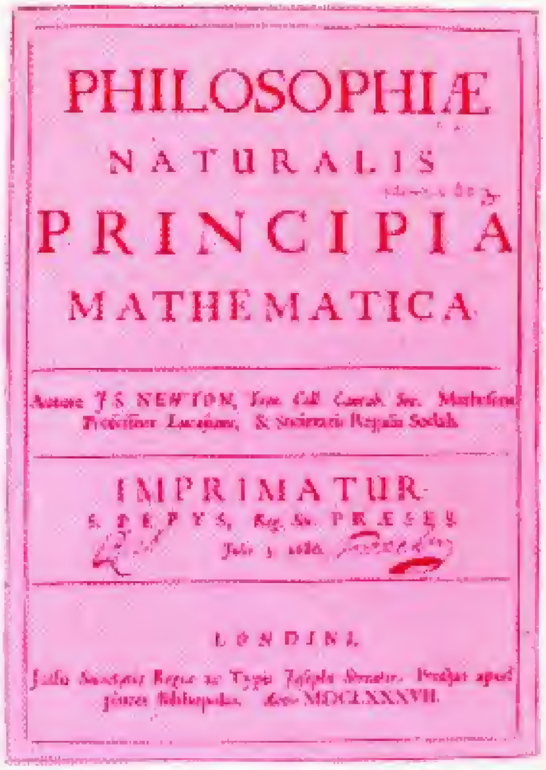

THE CASE OF THE
WALKING DOG
AN ISAAC NEWTON MYSTERY
BALAJI SAMPATH
With sections from
NEWTON'S ORIGINAL PRINCIPIA


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Whether you are a Physics student trying to figure out Newtonian Mechanics, or a science teacher looking for teaching tools or a layperson who wants to understand the basics of Physics, this cartoon dialogue book is just what you need.

What do Newton's three laws say about the world around us? Do these laws apply to human beings and dogs? This cartoon book explains it all. Learning Physics was never so easy!

Eureka Science Dialogue Series - 1

The Case of the Walking Dog
An Isaac Newton Mystery

Author: Balaji Sampath
Illustrations, Design: Basheer Ahmed and Marimuthu
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Foreword

As my car was cruising,
I looked at the speedometer
It was a steady 40 miles per hour
"Ah!" I said to myself,
My acceleration is zero
And by Newton's 2nd law
No forces are acting on me!

I turned the ignition off
to save some petrol,
since by Newton's 1st law
I'd move uniformly at 40 miles per hour
As no forces were acting on me!

The road was straight
And I dozed off to some music
when the radio woke me up --
"the traffic is held up
as some car has stopped
in the middle of the road..."
the radio blared.
I cursed the radio
and the driver of the
car that stopped...
Obviously it's someone who
doesn't know Newton's laws!

"Push your car off the road
by applying Newton's third law"
I screamed, "before my car hits
your car... At 40 miles per hour!!"

We all think we know Newton's laws, but do we really? Balaji Sampath's delightful book solves the mystery of the walking dog by taking us through a classroom where children with inquisitive minds are exploring with a competent teacher. Since Balaji bases the book on questions that were really asked by school children in urban and rural Tamilnadu in India, the book has an originality and freshness that would hardly have been thought possible for something on a topic where many text books have been written since ages.

The cartoons and discussions are funny and thought provoking and children, teachers and parents alike will enjoy while learning. By following the clues of this Isaac Newton Mystery, the readers will not only find out more about a curious dog but also will unravel everyday phenomenon and develop a mastery of school physics, where Newton's laws are applicable.

So without further ado turn the pages and you will continue to read until the book is over!

– Ravi Kuchimanchi

Ravi Kuchimanchi obtained his Ph.D. in particle physics from University of Maryland, College Park and did postdoctoral work in theoretical physics at University of Virginia, Charlottesville.

Preface

This book tries to explain Newton's Laws in simple terms. But this does not mean we sacrifice the insights and complexities of the subject for the sake of simplicity. Instead through a whole range of discussions and experiments, the key points are highlighted and the complexities described in detail.

This book claims to be simple and fun and yet meant for serious reading. It is meant for those who want to really understand what Newton's laws mean. Yet this also attempts to be a popular science book. Is it really possible to combine all these in one book? Only you - the reader - can tell.

Why Newton's Laws?

It is 400 years since these laws were written down. Today's fashion is writing on the meaning of quantum gravity, or black holes and big bang or even string theory. These are the latest developments in Physics. Popular Science books try to focus almost exclusively on these themes. So why have we brought out a book on good old Newton's laws, which every textbook states in 3 lines? For one, Newton's laws are well known but seldom understood. A complete understanding of Newton's laws is far more useful than learning half-baked quantum concepts that are thrown around in the middle of a conversation! Quantum mechanics is a wonderful subject - but it grows out of a through understanding of ordinary Newtonian Mechanics. Second, there is a lot that is exciting in Newtonian Mechanics - it *really* describes the world around us. A full understanding of Newton's laws can help you understand the things happening around us much better. In this book our attempt is not merely explaining what Newton said. We try to get readers to think like Newton, to get a feel for the scientific process of dialogue, hypothesis, experimentation and consolidation. We can learn names and concepts others have thought about. Or we can learn how to think about nature. In the final count the latter is always more rewarding and a lot more fun.

This is a dialogue book on Newton's Laws. What does that mean? The book is written as a classroom dialogue between students and a teacher. This is not an imagined dialogue. This book was developed by actually doing these activities and discussions in class. The questions you see here are *really* questions that students raised in class when we had these discussions. This material has been tested in a whole range of schools - from the poorest schools to the most elite schools, and even with 3rd year B.Sc. Physics students in colleges. The surprising thing was the response to this material across this wide spectrum was pretty much the same - the amount of fun the students had thinking about these otherwise "boring" laws is what

encouraged us to come out with a book.

After this dialogue was written, other volunteers used it and provided their inputs. Based on their suggestions, new chapters clarifying difficult areas were introduced. This book weaves together *real* questions, doubts and problems all these people from many different and varying backgrounds had on this subject.

The first chapter is very easy. Almost everyone can read it without much effort. The second chapter is slightly harder - it will take a few readings to completely grasp the arguments. The third chapter requires a bit of high school mathematics and introduces detailed problem solving techniques. There is a note at the end of chapter 3 which introduces the reader to vectors. This note will be useful in understanding the problem solving techniques introduced in chapter 3. Finally we have sections from Newton's Original Principia as an Appendix. This is not a book that can be read from beginning to end like a novel. You have to read it once and then re-read each chapter two to three times to really understand all the points.

Science books usually state facts with absolute certainty. In this book, we instead, point out problems with the facts, find holes and make these laws sound uncertain. Unfortunately our society fears uncertainty. This fear drives us to ideas that sound certain - even if they are false. But to do good science, you must overcome the fear of uncertainty. The instant clarity you get by merely accepting a fact without thinking hard about it is not real clarity. If you feel uncertain, it just means you are thinking about the problem, turning it around in your head, trying to internalize it and link it with what you already know. This is a sign of good science! Truth is more important than certainty.

There are 3 groups which can use this book:

1. **Students** - You know Newton's laws already - but do you really understand what it means? If you don't see Newton's Laws in everything that happens around you, then you need this book!
2. **Science Teachers** - Many good teachers want to convey the spirit behind a subject. But limited time and a lot of topics to cover make this hard. What is the real meaning of Newton's Laws, how to introduce it in class, how to get students to actively discuss these laws - as teachers all these are important questions that you need to answer. This book will hopefully help you in this process.
3. **Others** - You probably learnt and promptly forgot Newton's Laws in school. This book hopes to get you thinking about forces and motion - about what you see around you and about the basic process of scientific thinking.

Why Dialogue?

I have found science dialogues are the best way to get a class actively interested in the subject. As you try these in class, I am sure you will agree. Dialogue and debate is the way of science - and as we do it we learn more than just the facts. We learn how to think - which is after all what real learning is.

In the usual mode of learning, a teacher shows us the 'right' way to do things. The 'right' method is repeated several times until we forget our old 'wrong' way of doing things. The problem with this approach is that the learner seldom figures out what was wrong with his/her original way of thinking. The new idea is uncritically accepted - but the old idea remains in our head along with the new ideas. This creates conflicts and a very confused understanding. The student is very unsure of what he/she knows and is not able to figure out what he/she understands and what he/she does not. This leads to a serious lack of confidence.

I understand an idea better only when I figure out what was wrong with my earlier approach. Only by figuring out what is 'wrong' with our 'wrong' ideas can we figure out what is 'right' with the 'right' ideas! This makes our understanding sure-footed and thorough. Learning what is wrong with our 'wrong' ideas requires a dialogue - merely stating what is right is not enough. A dialogue can bring out different perspectives and discuss why some perspectives are better than others and what is wrong with the other perspectives.

Dialogue is fundamental to science. Science grows through dialogues. Raising and answering questions, clarifying concepts, doubting claims, offering alternatives and devising experiments - all this is what constitutes active science. This is the best way science can be learnt. This is the philosophy behind the Eureka Science Dialogue Series.

Eureka Science Dialogue Series

Eureka Science Dialogue Series is an attempt to bring out a set of books to introduce basic concepts through simple discussions. The first in this series is this dialogue on Newton's Laws. We believe that everyone can understand our world. That this is not the case today is primarily because we have not made the effort to explain things simply enough. We also believe that a better understanding of the world around us will help us develop into more confident and capable people.

Eureka Science Education Network is a joint programme of the Tamilnadu Science Forum and the Association for India's Development. The Tamilnadu Science Forum is a voluntary organization working on literacy, health, education and women's empowerment in villages, slums and schools all over Tamilnadu. Association for India's Development is a network of Indian students and professionals in software companies, research institutes and universities all over the world who work on education, health and rural development issues. At the end of this book is a brief note on Eureka Science Education Network, Tamilnadu Science Forum and Association for India's Development.

- Balaji Sampath
2nd October, 2002

Although the whole of philosophy is not immediately evident, still it is better to add something to our knowledge day by day than to fill up men's minds in advance with the preconceptions of hypotheses.

- Isaac Newton

Acknowledgments

There is a long list of people I need to acknowledge. The many schools, teachers and children who participated in the dialogue classes should be thanked - P.S. Senior, DAV, Sarada School, Shankara school, Rani Meyammai, St. Ebba's, Ahobilam Oriental and Nugambakkam Corporation schools. I would like to also thank the Physics Department of the Stella Maris College for a session with the B.Sc. Students.

I have benefited greatly from long discussions with Prof. K.S. Balaji. Prof. K.S. Balaji and I jointly teach Physics for 11th and 12th students who want to get into the IITs. After every class, we discuss questions raised by students and spend a lot of time trying to figure out better ways of communicating the joy of doing Physics. I would like to particularly thank a few of my students - Niranjan, Anuthama, Naresh, Karthik and Vikram - with whom I had great fun developing this dialogue class.

When we look back, there are a few teachers who stand out - who have played an important role in shaping our thoughts. Prof. Ramabadran was one such teacher. It is from him that I learnt all the Physics that I know - particularly key to this book were the dialogue sessions I had with him on Newton's Laws. I had told him a year earlier that I was bringing out this book and he was very happy. His recent demise has been a shock to me and to many others who learnt our physics from him.

Ravishankar helped with editing the initial draft - proof-reading, correcting, re-writing and suggesting cartoons. Without his help this book wouldn't have looked like this!

I would like to thank a number of TNSF activists and teachers who read the initial drafts and provided valuable comments. Of these I particularly thank Archana, Hemavathy, Gomathy, Suresh, C. E. Karunakaran and Ganesh. The constant encouragement from Dr. Sundararaman and Aruna has been a great support. Of course without the continuous support and love from my wife Kalpana, writing this book would not have been as much fun.

I particularly want to thank Dr. Ramanujam who started me off working in schools and in particular developing dialogue classes. Dr. Ramanujam, a computer scientist at the Institute of Mathematical Sciences has been an inspiration to all young TNSF volunteers. His work with TNSF and in education has opened up new ways of looking at the education system and classroom interactions. This whole idea of doing science dialogues in schools and particularly this Newton's Law Dialogue owes a lot to him.

And finally I would like to thank the Science Movement for giving me an opportunity to work with schools and school teachers in such large numbers. This book rightly belongs to the activists and teachers in the science movement who will hopefully find this a useful tool in improving the quality of middle and high school science education.

This book is dedicated to

Prof. Ramabadran

and to teachers like him
who inspire in students
an undying love
for their subject.

Newton's Three Laws of Motion

Newton wants to ***calculate and predict*** how objects move. He does this by setting out three laws that all objects obey. First he talks about the natural motion of an object.

Newton's First Law: A body at rest continues to remain at rest and a body in motion keeps on moving uniformly with a constant speed in a straight line if there is no net external force on the body.

Therefore being at rest and moving uniformly forever are two possible natural motions for every object.

Newton then discusses why sometimes objects differ from their natural motion. This he says happens because other objects influence or 'act' on this object. The way they act on this object is by exerting a 'force' - a push or a pull. This force changes the way the object moves. Since objects move with constant velocity even when there is no force, force does not cause velocity. Instead force causes a change in velocity. The rate of change of velocity is acceleration and force is proportional to this.

Newton's Second Law: F=ma. The net external force on an object changes the object's motion and the net external force is equal to the rate of change of momentum of the object. For most simple cases, this law can be stated as **$F_{external}$ = mass x acceleration.**

This is the famous F = ma. With the first two laws, Newton has given us a programme to calculate how things move. Look at how the object moves in the beginning. Get its velocity. With this you can calculate the new position. Use the force on it to calculate the acceleration. With this acceleration, you can calculate the new velocity and with that you can calculate the next position, and so on. This is the basic Newtonian programme for understanding motion - at least in principle. What is left is actually finding out what the forces on objects are. This turns out to be quite tough. Newton gave us the exact force law in the case of gravitation: $F = (Gm_1m_2)/r^2$. With this we can calculate the motion of planets and comets. He then gave a general law about forces. This doesn't tell us what the force law exactly is, but tells us what one force should be in terms of another.

Newton's Third Law: For every action there is an equal and opposite reaction. Whenever a body A exerts a force on another body B, the second body B exerts an equal and opposite force on body A.

This law merely states as a principle the observation that whenever you press something, that something presses you back equally. Action and reaction are just forces - and they act together. They are not cause and effect. Newton tells us that if you know one force, you also know the other. Newton then proceeds to show how a lot of things around us can be explained using these three basic laws.

To the Teacher...

Newton's laws can be stated very easily. But understanding them is an entirely different matter. In working with school children, we found a lot of misconceptions around the meaning of force and Newton's laws. For example...

1. There is a belief that these laws work only for non-living things.
2. There is a lot of confusion about the words action and reaction.
3. There is also a lot of confusion between the concepts of pushing and moving.
4. There isn't sufficient understanding of what uniform motion forever means.
5. These laws are seen as three different independent statements and not as an integrated package that helps us understand how the world works.
6. Because students haven't applied these laws in real life situations, these laws seem very *'text-book-ish'* - therefore students inside their heads assume these laws are meant for answering questions and not for thinking about the world.

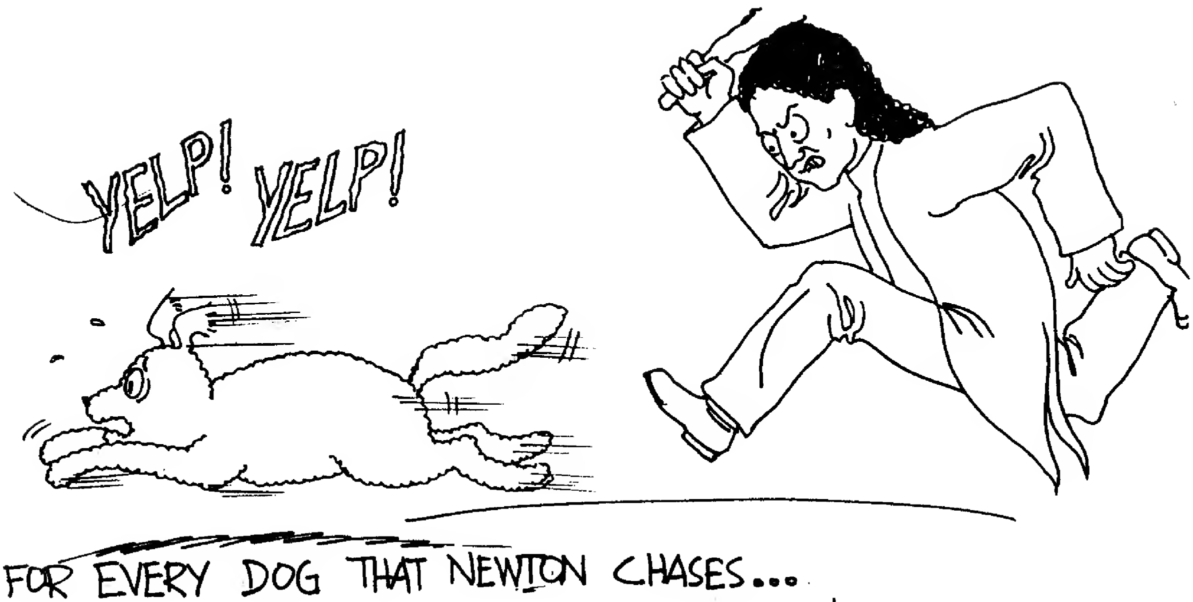
In this book, we look at classroom discussions that bring out questions in students' minds. We have found such discussions very rewarding - often for the first time, students begin to believe in the reality of these laws and begin to apply these laws to the world they see around themselves.

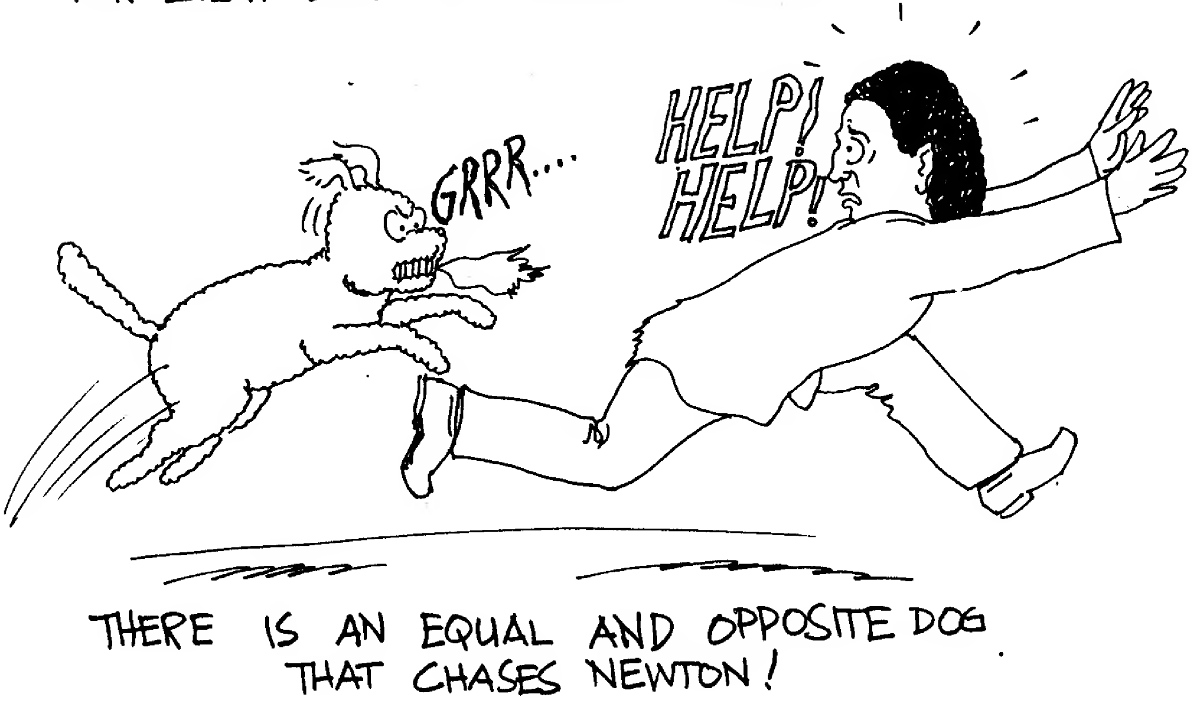
The main idea that must be conveyed to students is that these laws form **one single complete package**. It is not law number 1,2,3,4,... They have to see how these laws together help us understand and explain the world. These laws are after all statements gathered from practical observation of things around us. They offer us a world-view - a way of thinking about the world. This world-view is what students have to grasp. Understanding each law separately will just not do. Understanding the framework behind Newton's laws and how within this framework these laws help us ask questions about motion and helps us answer them quantitatively - this is what is needed.

Newtonian framework is an excellent introduction to scientific thinking. A framework that shows how the world works. A framework that suddenly makes a lot of inexplicable things around us seem explainable. Introducing the student to this way of thinking can put him or her on the path to a scientific outlook on life. If this can be done, this book would have achieved its purpose.

Chapter 1

Isaac Newton's Dog and His Three Laws





Chapter 1: Section A

Dog at rest does not remain at rest !

There are two parts to Newton's first law. A body at rest continues to be at rest *and a body* that is moving keeps on moving forever - if there is no external force.

In this section, we look at the first part of this law. The law in this form looks very simple. Bodies at rest remain at rest if you don't disturb them.

Most people would accept this as a simple law - just stating a daily observation that undisturbed, things remain at rest. But if you accept it as such, then you also observe that a sleeping dog gets up and starts walking all on its own without being disturbed. Now both these observations are not consistent. So most people sub-consciously conclude inside their heads that this first law works only for non-living things.

But the point is that Newton's first law is a statement for all bodies - not just non-living things. In this section we use examples to point out how this law seems to be violated in a number of cases. We then try to argue what Newton really meant by the first law.

In this section our aim is to confuse you! We want to shake your belief in the idea that bodies at rest remain at rest if they are not disturbed. Let's see how well we succeed!

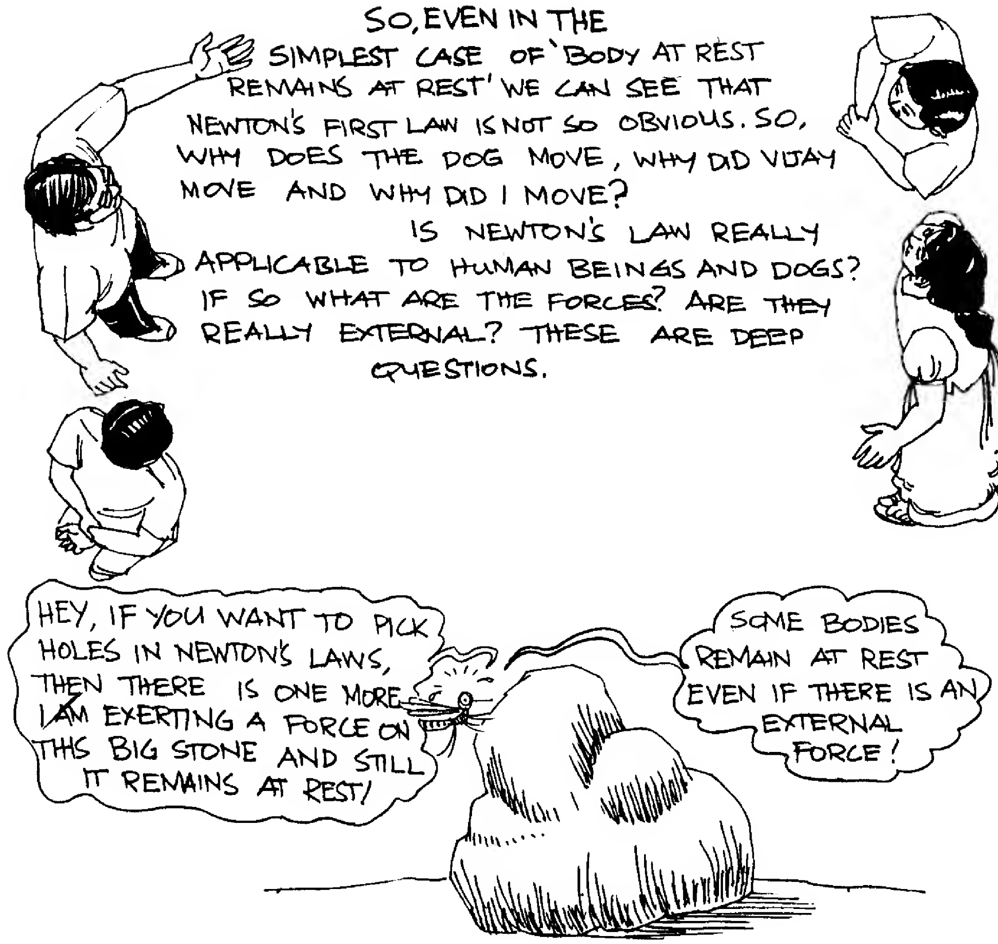

CAN SOMEONE TELL ME WHAT IS NEWTON'S FIRST LAW?
EVERY BODY CONTINUES TO BE IN A STATE OF REST OR OF UNIFORM MOTION IN A ST STRAIGHT LINE TILL AN EXTERNAL FORCE ACTS ON IT!
THIS SOUNDS COMPLICATED AND IN FACT A LITTLE TOO STRICT FOR ALL PARTICLES TOOBEY!
?
WHAT DO YOU THINK WILL HAPPEN IF A BODY VIOLATES THE LAW? WILL IT BE PUNISHED?


DO YOU THINK ANYBODY WILL DARE VIOLATE THIS LAW?
NO!
THEN LET US UNDERSTAND WHAT A BODY SHOULD DO TO AVOID PUNISHMENT. WHAT ARE THE TWO THINGS THAT SHOULD HAPPEN IF THERE IS NO EXTERNAL FORCE?
A BODY AT REST CONTINUES TO BE AT REST AND A BODY THAT IS MOVING CONTINUES TO MOVE UNIFORMLY IN A STRAIGHT LINE.
A BODY AT REST CONTINUES TO BE AT REST. IS THIS TRUE FOR ALL BODIES? DOES IT WORK FOR THIS SLEEPING DOG?
Z!
YES!
YES!
LET ME THROW A STONE AT THE DOG!


SEE, THE DOG WAKES UP AND RUNS. WHERE IS THE EXTERNAL FORCE?
DOG IS A LIVING THING IT CAN THINK!
GRR...
OK! SO YOU ARE SAYING NEWTON'S LAWS ARE NOT APPLICABLE TO DOGS. TAKE OUT YOUR PHYSICS TEXT BOOK. READ OUT NEWTON'S FIRST LAW FROM THE BOOK.
EVERY BODY AT REST...
ENOUGH ... ENOUGH... TAKE OUT YOUR PENS AND MAKE A CORRECTION - INSTEAD OF "EVERY BODY AT REST" MAKE IT "EVERY BODY WHICH IS NOT A DOG AND WHICH IS AT REST..."
HA! HA!
GRR.... WAKING ME UP JUST TO TALK ABOUT SOME NEWTON FELLOW!


SOCIETY FOR PROMOTION OF DOGISH WELFARE
GRRR
THIS IS TOO MUCH! DOG'S HAVE RIGHTS WE CAN'T BE USED AS EXPERIMENTS.
OOH!
AH!
AAH! HA!
OK! THAT DOG DOESN'T WANT US TO EXPERIMENT WITH HER! WE WILL USE YOU AS AN EXAMPLE.
VIJAY! STAND STILL. NOW WALK FROM HERE TO THERE.
VIJAY WAS FIRST AT REST AGREED? HE MOVED FROM HERE TO THERE. NEWTON SAYS HE CAN'T MOVE IF THERE IS NO EXTERNAL FORCE...
WHAT WAS THE FORCE ON HIM?
17


YOU TOLD HIM TO MOVE!
SO, YOU SAY MY TELLING HIM IS A FORCE. OK, NOW I AM GOING TO MOVE FROM HERE TO THERE!
NOW WHAT WAS THE FORCE ON ME? NO ONE TOLD ME TO MOVE!
YOUR MIND TOLD YOU TO MOVE. THAT'S WHY YOU MOVED!
SO MY MIND IS THE EXTERNAL FORCE MOVING ME. BUT IS MY MIND EXTERNAL OR INTERNAL TO ME? THE LAW REQUIRES AN EXTERNAL FORCE. SO HOW CAN MIND-WHICH IS INTERNAL TO ME-MAKE ME MOVE? ALSO WE WANT A 'FORCE' -DO YOU REALLY THINK 'MY ASK -ING YOU' IS A FORCE? SUPPOSE I ASK YOU, AND YOU DON'T MOVE?



Summary so far...

Newton's First Law has two parts:

1. A body at rest continues to be at rest.
2. A body in motion keeps on moving

... if there is no external force on the body.

The second part of this law does not seem very obvious. So we decided to come back to that later. But the first part seems obvious. But when we try to see if it works for dogs or human beings, we get into trouble. So now the first part of this law does not seem obvious either!

Chapter 1: Section B

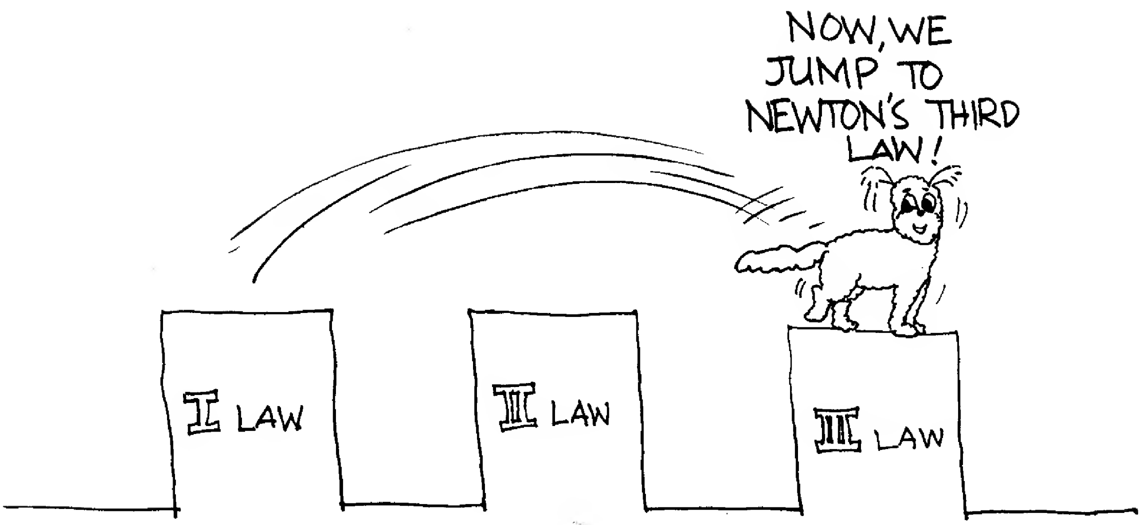
What is your reaction when I slap you ?

To satisfactorily answer why the dog can walk and what the forces on the dog really are, we will need to understand Newton's 3rd Law. One usually presumes that one can understand these three laws in order - first law first, then the second law and then the third law. But this is not true. The three laws form one single package. You need the third law in order fully understand the first law and you need the first law to completely understand the second law. These laws are one integrated whole - they form a framework for understanding motion. To get at this framework we will have to go back and forth between these three laws several times. So in this section we will now jump to the Third Law. Once we get a fair understanding of the third law, we will then come back in the final section to explain why the dog can walk.

The third law says that for every action there is an equal and opposite reaction. Most people understand this to mean "if something happens, it will cause something else." Since this statement looks so simple, this is one of the few laws everyone remembers even many years after school. Stating this law is easy. But getting at its real meaning is not that trivial. A deep understanding of this law is essential to understanding Newton's framework for motion.

This section looks at the meaning of Action and Reaction. We start with a simple case of a student being slapped and ask what is the reaction. We see that things like pain, crying, slapping back etc cannot be the reactions we want. Finally we arrive at the idea that in physics, Actions and Reactions are just forces - not 'doings'.

Another common confusion is between the ideas - 'pushing' and 'moving'. Getting the difference between them is the key to understanding the third law. Once we make this distinction, we move on to quantifying the *amount of* force and see that to exert a force we need something that will resist our application of force. This then is the observation that Newton states as his third law. Having understood this, the reader is then led to the resolution of the slapping example.


NOW, WE JUMP TO NEWTON'S THIRD LAW!
I LAW
II LAW
III LAW


LET'S GET ON WITH UNDERSTANDING OR RATHER CONFUSING NEWTON'S THIRD LAW
CAN SOMEONE STATE THE THIRD LAW?
FOR EVERY ACTION, THERE IS AN EQUAL AND OPPOSITE REACTION.
AH! AT LEAST NO DOGS IN THIS ONE!
21

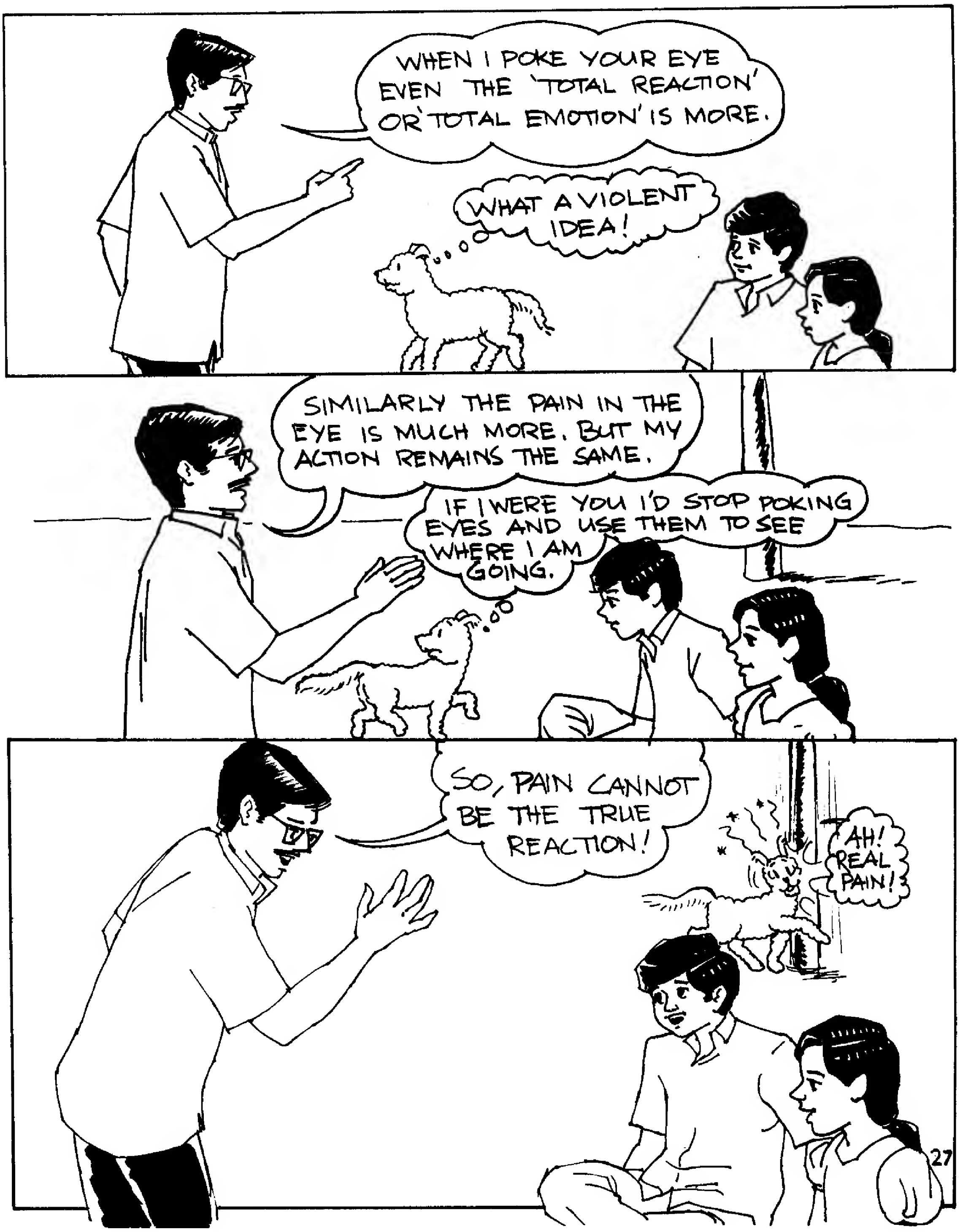
EXCELLENT! NOW COME HERE RAMYA. I AM GOING TO SLAP YOU. DON'T WORRY- ONLY LIGHTLY!
NOW WHAT IS THE ACTION HERE?
YOUR SLAPPING
OK, WHAT IS THE REACTION?
ACTION - REACTION
SLAPPING
TURNING HER FACE!
PAIN FELT BY HER CHEEK
PAIN FELT BY THE HAND!
SOUND!
IS THAT ALL? WHAT IF I SLAP RAMYA HARDER?
CRYING. HAND MARKS ON HER FACE
WHAT IF I AM NOT YOUR TEACHER? YOU WILL SLAP ME BACK. WON'T YOU? SO, GO AHEAD AND SLAP ME BACK.
AFTER A LOT OF COAXING RAMYA SLAPS ME!
SO, DO YOU HAVE ONE MORE REACTION?

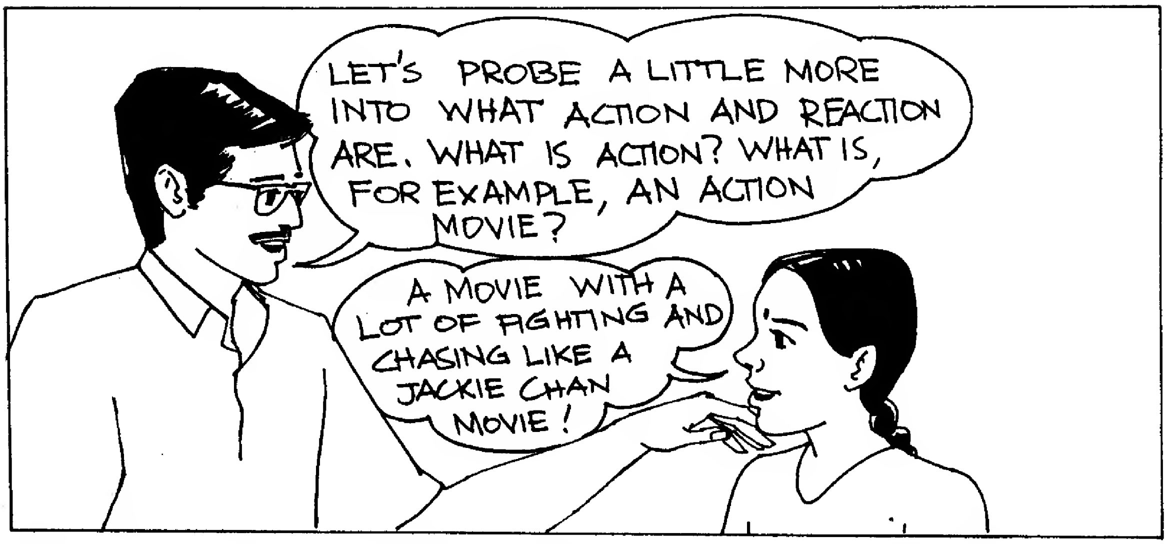
YES. SLAPPING BACK
ANGER IS ALSO ANOTHER REACTION
YES - ANGER AND POSSIBLY SHAME AND OTHER EMOTIONS. WE WILL CLUB IT AS EMOTIONS.
ACTION
REACTION
SLAPPING
TURNING HER FACE
PAIN IN CHEEK
PAIN IN HAND
SOUND
CRYING
MARKS ON FACE
SLAPPING BACK
YOU HAVE A WHOLE LOT OF REACTIONS AND THE ACTION ALL LISTED HERE. THEY ARE ALL REACTIONS - AGREED? NEWTON'S LAW SAYS FOR EVERY ACTION THERE IS AN EQUAL AND OPPOSITE REACTION. NOW TELL ME HOW IS CRYING EQUAL AND OPPOSITE TO SLAPPING?
THE HARDER YOU SLAP THE MORE SHE WILL CRY.
ZZ

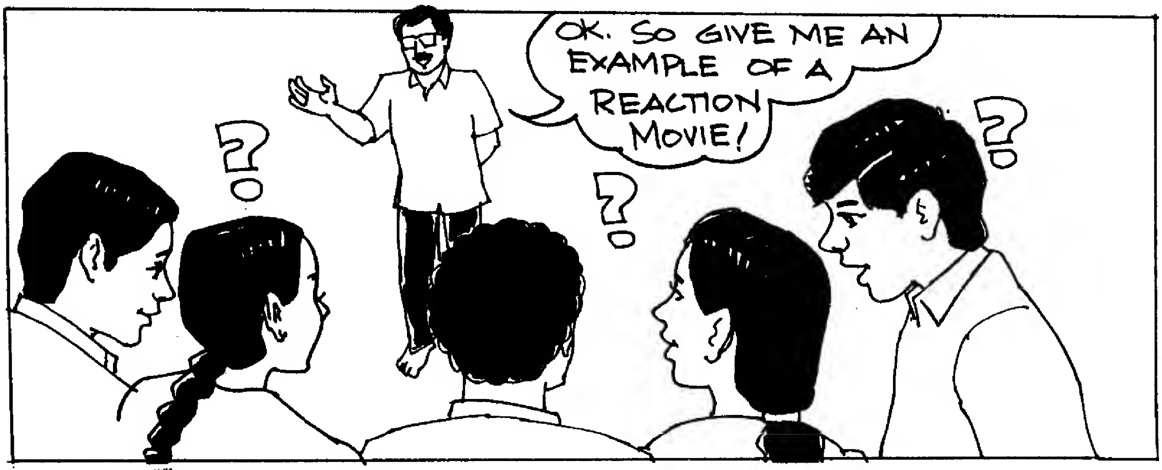
BUT RAMYA WON'T CRY THE SAME EVERYTIME I SLAP HER THE SAME WAY. IN FRONT OF THE CLASS, SHE DID NOT EVEN CRY. BUT IF I SLAPPED HER WHEN SHE IS ALONE, MAYBE SHE WOULD HAVE. AND IF I SCOLD HER AND THEN SLAP, SHE WILL CRY A LOT. SO CRYING CAN'T BE EQUAL TO SLAPPING ALWAYS.
ANYWAY WHAT DOES IT MEAN TO SAY `CRYING IS EQUAL TO SLAPPING? WHEN CAN WE SAY TWO THINGS ARE EQUAL? ONLY WHEN THEY ARE OF THE SAME TYPE - ISN'T IT? CRYING OR TURNING FACE IS NOT LIKE SLAPPING. SO HOW CAN IT BE EQUAL?
SO, SLAPPING BACK MUST BE THE REACTION - AND IT IS EQUAL AND OPPOSITE.
IT IS GOOD TO SEE HUMANS SLAPPING EACH OTHER INSTEAD OF THROWING STONES AT US.
SOCIETY FOR PROMOTION OF DOGISH WELFARE


OK. BUT IF YOUR MOTHER OR TEACHER SLAPS YOU, WILL YOU SLAP BACK? WHAT IF A FRIEND SLAPS YOU? SOMETIMES YOU SLAP BACK. SOME -TIMES YOU DON'T. SO, SHALL WE CHANGE NEWTON'S THIRD LAW? CHANGE THE SENTENCE IN YOUR BOOKS - INSTEAD OF 'FOR EVERY ACTION THERE IS ALWAYS EQUAL AND OPPOSITE REACTION' MAKE IT 'FOR EVERY ACTION THERE IS SOME -TIMES AN EQUAL AND OPPOSITE REACTION.
...OR IS NEWTON SAYING 'IF YOU SLAP ME TODAY, SOMETIME TODAY, TOMORROW OR ATLEAST IN MY NEXT LIFE, I WILL DEFINITELY SLAP YOU BACK!
HA... HA...
OK, ENOUGH JOKES. NOW WE ALL KNOW WE CANNOT CHANGE NEWTON'S LAWS. LET'S TRY TO UNDER STAND THE QUESTION AGAIN. I HAVE WRITTEN ALL THESE THINGS ON THE BOARD. ARE THEY ALL REACTIONS? IS ANGER A REACTION? IF SOMEONE IS ANGRY DON'T YOU SAY 'HE IS REACTING TO BEING SLAPPED?' IF ANGER IS 'REACTING TO' IS IT NOT A REACTION?

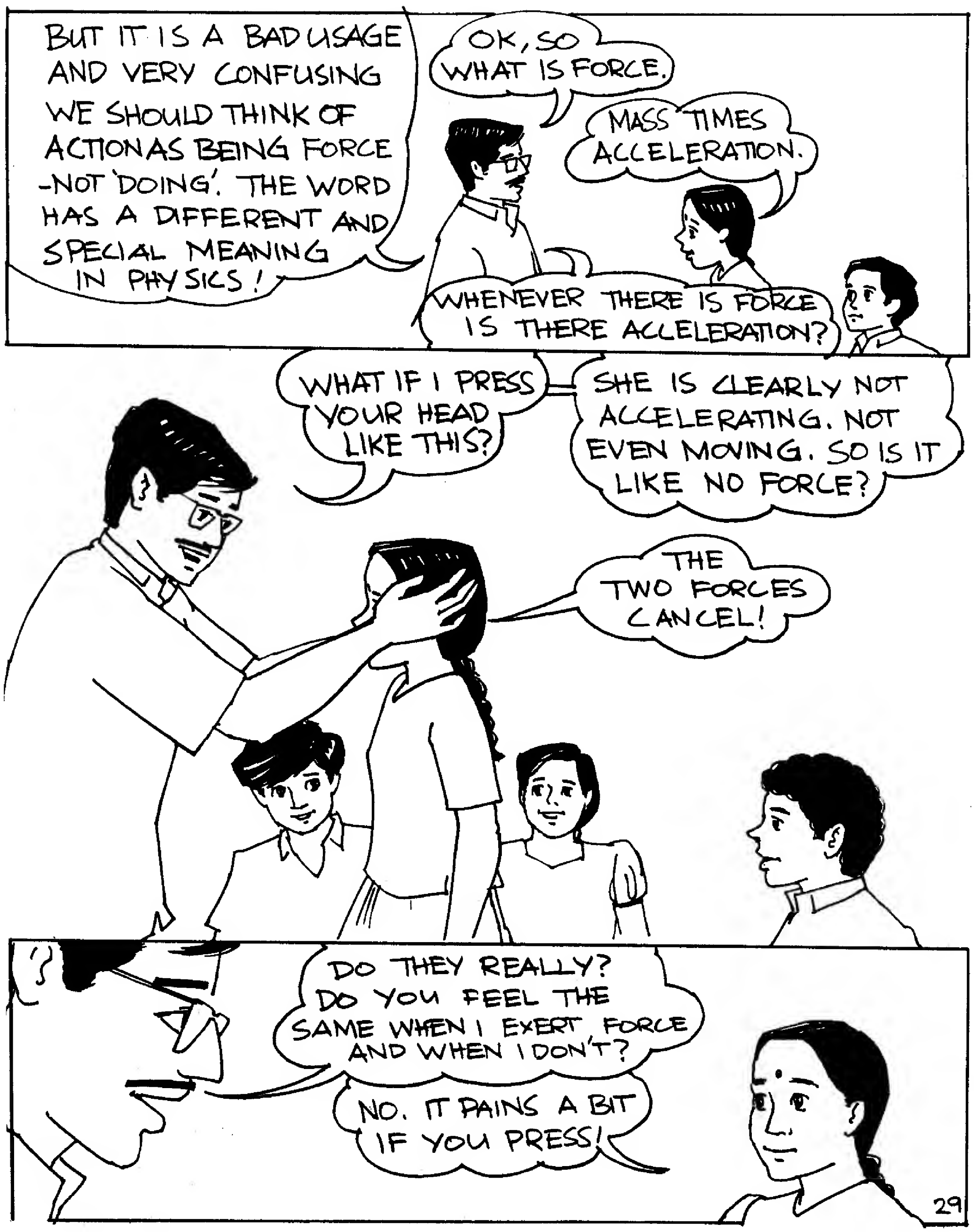

ALL THESE ARE REACTIONS - BUT ONLY ONE OF THEM IS NEWTON'S REACTION. THE PAIN FELT BY THE CHEEK. ONLY THIS HAPPENS ALWAYS.
NEWTON'S LAW SAYS THAT THE TOTAL EMOTION FELT BY RAMYA IS THE EMOTION WITH WHICH YOU SLAPPED.
BOTH THESE ARE VERY INTERESTING IDEAS. WE SHOULD UNDERSTAND THESE BETTER. THE SECOND IDEA POINTS OUT THAT SOMEWHERE DEEP WITHIN US WE BELIEVE THAT THERE IS A 'NET OR TOTAL REACTION' WHICH IS EQUAL AND OPPOSITE. THE FIRST IDEA THINKS IN TERMS OF SEVERAL REACTIONS - ONE OF WHICH CAN SATISFY NEWTON'S LAW IN EACH CASE.
THINK OF MY POKING YOUR EYE WITH THE SAME AMOUNT OF 'ACTION' AS MY SLAPPING YOUR CHEEK. WHEN IS YOUR REACTION MORE?
WHEN POKING MY EYE!


WHEN I POKE YOUR EYE EVEN THE 'TOTAL REACTION' OR 'TOTAL EMOTION' IS MORE.
WHAT A VIOLENT IDEA!
SIMILARLY THE PAIN IN THE EYE IS MUCH MORE. BUT MY ACTION REMAINS THE SAME.
IF I WERE YOU I'D STOP POKING EYES AND USE THEM TO SEE WHERE I AM GOING.
SO, PAIN CANNOT BE THE TRUE REACTION!
AH! REAL PAIN!



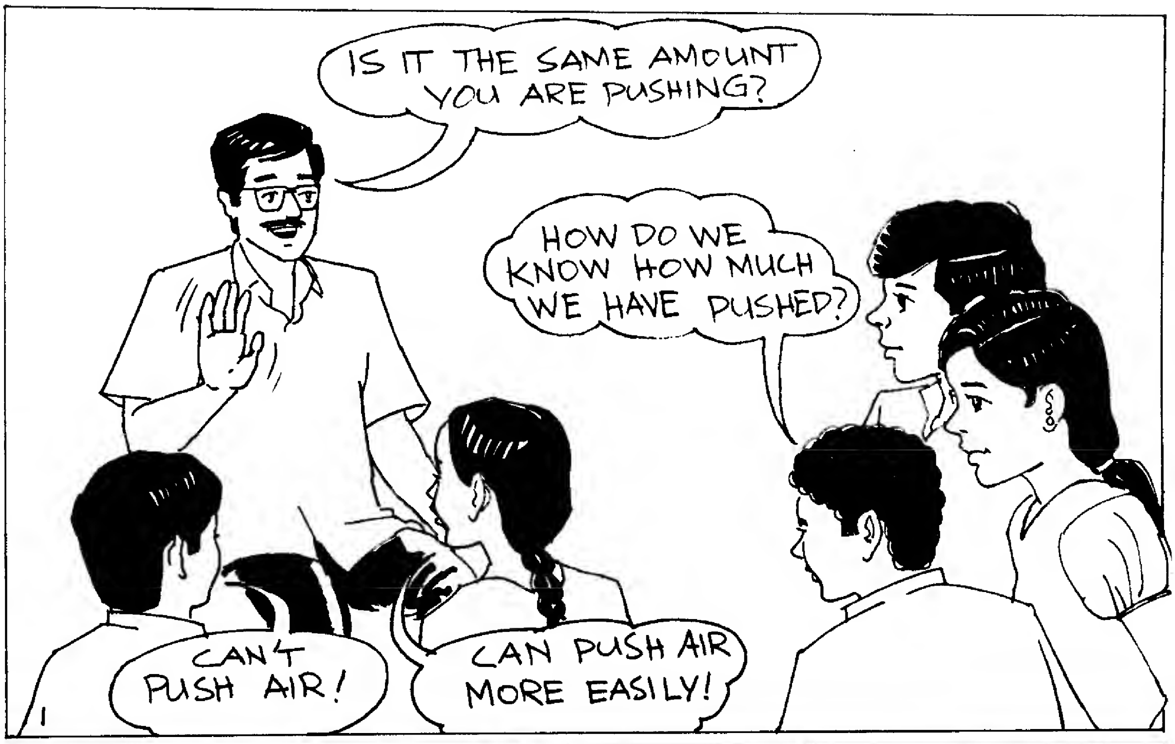


We have to be careful when we use these words. The everyday usage of the word Action, is not what Newton meant. In Physics, we use words in special ways. By Action, Newton meant Force. (See Newton's Original Principia in the Appendix). Newton's Real Law is **Force exerted by one body on another is equal and opposite to force exerted by the second body on the first**. The words action and reaction sound nice...



BUT IT IS A BAD USAGE AND VERY CONFUSING WE SHOULD THINK OF ACTION AS BEING FORCE -NOT 'DOING'. THE WORD HAS A DIFFERENT AND SPECIAL MEANING IN PHYSICS!
OK, SO WHAT IS FORCE.
MASS TIMES ACCELERATION.
WHENEVER THERE IS FORCE IS THERE ACCELERATION?
WHAT IF I PRESS YOUR HEAD LIKE THIS?
SHE IS CLEARLY NOT ACCELERATING. NOT EVEN MOVING. SO IS IT LIKE NO FORCE?
THE TWO FORCES CANCEL!
DO THEY REALLY? DO YOU FEEL THE SAME WHEN I EXERT FORCE AND WHEN I DON'T?
NO. IT PAINS A BIT IF YOU PRESS!

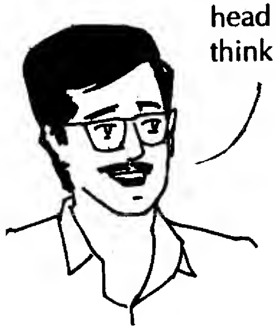

SO TWO EQUAL AND OPPOSITE FORCES DON'T ALWAYS CANCEL. IT IS NOT THE SAME AS ZERO FORCES. TWO FORCES ONLY CANCEL IF YOU WANT TO LOOK AT NET MOTION. AND FORCES CAN DO OTHER THINGS APART FROM MOVING THINGS. IF YOU EXERT FORCES ON A SPRING - IT WON'T NECESSARILY MAKE IT MOVE. IT WILL COMPRESS THE SPRING INSTEAD. FORCES CAN DO MANY THINGS - MOVING IS JUST ONE OF THEM.
SO WHAT IS FORCE?
IT IS A PUSH
OK. FORCE IS SIMPLY A PUSH NOW PUSH THE EARTH IN FRONT OF YOU!
PUSH IT DOWNWARDS... COME ON - PUSH HARDER OK ENOUGH. YOU KNOW HOW MUCH YOU PUSHED.
OK ENOUGH. NOW PUSH THE AIR IN FRONT OF YOU WITH THE SAME FORCE!
30


IS IT THE SAME AMOUNT YOU ARE PUSHING?
HOW DO WE KNOW HOW MUCH WE HAVE PUSHED?
CAN'T PUSH AIR!
CAN PUSH AIR MORE EASILY!

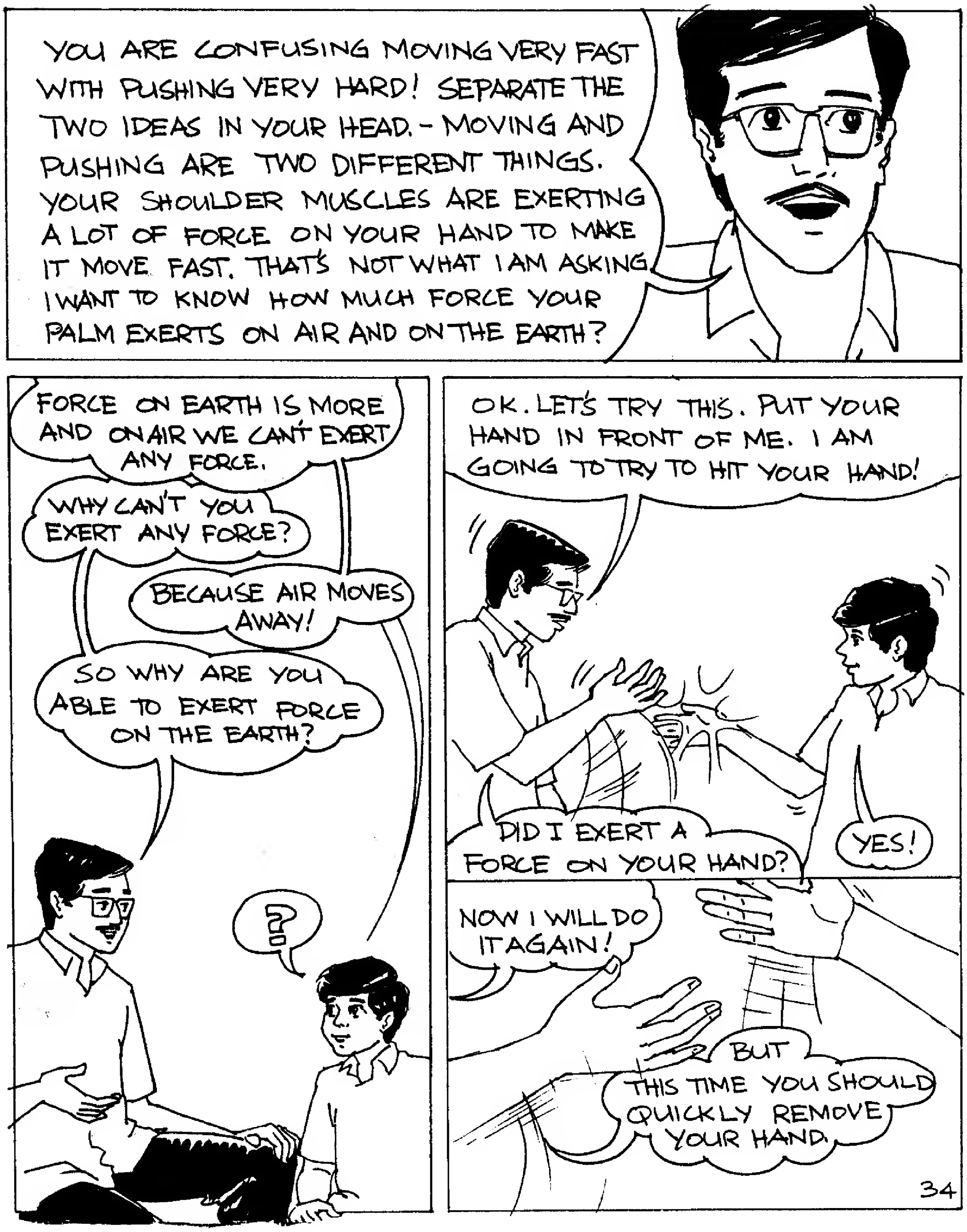

CAN YOU PUSH THAT LAMP POST?
NO. WE CAN'T PUSH IT.
REMEMBER THERE ARE TWO WORDS WE USE ALMOST INTER CHANGEABLY. 'CAN YOU MOVE THE LAMP POST' IS ONE. 'CAN YOU PUSH THE LAMP POST' IS A DIFFERENT ONE. WHICH ONE CAN'T YOU DO?
OH! WE CAN PUSH! WE CAN'T MOVE THE LAMP POST!

Somewhere in our minds we confuse two ideas (and two words) because they seem related and often happen together. PUSH and MOVE are classic examples. In science and **particularly in Physics,** we have to be careful about our own ideas. Physics is not about learning other people's ideas - it really is about questioning your own ideas and understanding them better.

Remember to keep this distinction in mind about the difference between push and move. PUSH is what **you** do. MOVE is what **it** does. It moves **because** you push - not the other way around. This mixing of ideas in your head will keep cropping up again and again and you will have to patiently think through and separate the two ideas.



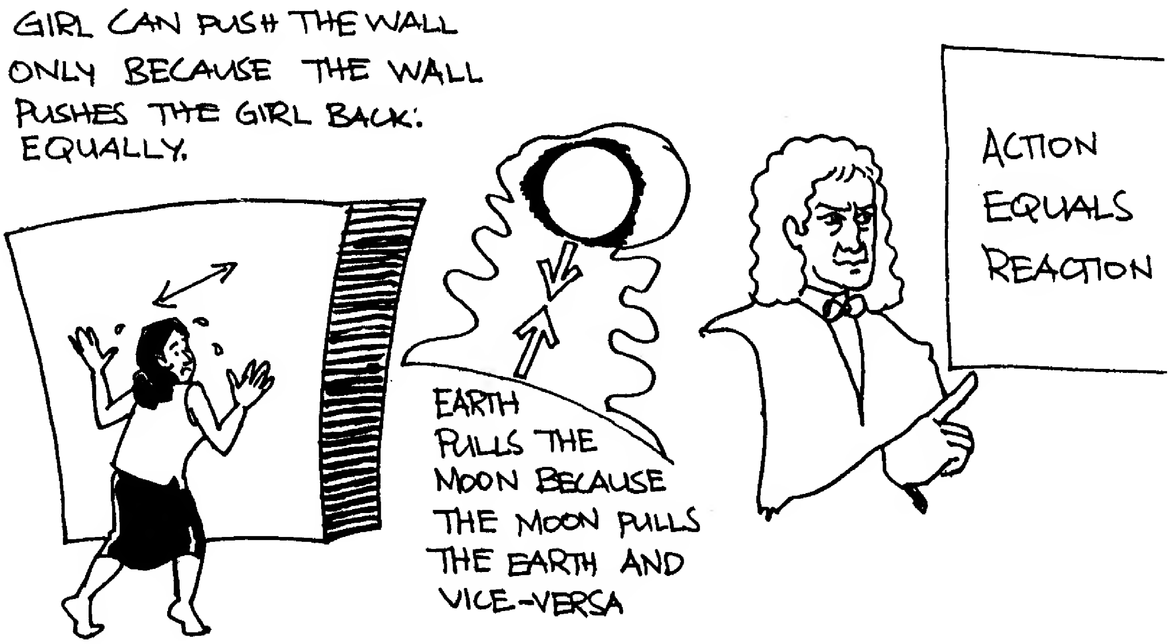

CAN YOU PUSH AIR WITH THE SAME FORCE?
YES!
NO!
NO!
YES!
YES!
LET'S VOTE. HOW MANY SAY YES?
HOW DO YOU KNOW YOU ARE EXERTING THE SAME FORCE?
SEE! WE ARE PUSHING THE SAME AMOUNT!


YOU ARE CONFUSING MOVING VERY FAST WITH PUSHING VERY HARD! SEPARATE THE TWO IDEAS IN YOUR HEAD. - MOVING AND PUSHING ARE TWO DIFFERENT THINGS. YOUR SHOULDER MUSCLES ARE EXERTING A LOT OF FORCE ON YOUR HAND TO MAKE IT MOVE FAST. THAT'S NOT WHAT I AM ASKING I WANT TO KNOW HOW MUCH FORCE YOUR PALM EXERTS ON AIR AND ON THE EARTH?
FORCE ON EARTH IS MORE AND ON AIR WE CAN'T EXERT ANY FORCE.
WHY CAN'T YOU EXERT ANY FORCE?
BECAUSE AIR MOVES AWAY!
SO WHY ARE YOU ABLE TO EXERT FORCE ON THE EARTH?
?
OK. LET'S TRY THIS. PUT YOUR HAND IN FRONT OF ME. I AM GOING TO TRY TO HIT YOUR HAND!
DID I EXERT A FORCE ON YOUR HAND?
YES!
NOW I WILL DO IT AGAIN!
BUT THIS TIME YOU SHOULD QUICKLY REMOVE YOUR HAND.


WHAT FORCE DID I EXERT ON YOUR HAND?
NOTHING.
WHY?
BECAUSE THERE WAS NO HAND TO STOP YOU!
THATS EXACTLY THE POINT! I CAN EXERT A FORCE ON YOUR HAND ONLY IF YOU STOP ME - IF YOU RESIST ME. THE EARTH RESISTS MY MOVING DOWN AND THEREFORE I EXERT A FORCE ON IT. YOUR HAND TRIES TO STOP ME. SO I EXERT A FORCE ON IT. MY ABILITY TO EXERT A FORCE ON YOU COMES FROM YOUR RESISTANCE.
THE STONE EXERTED A FORCE ON YOU BECAUSE YOU EXERTED A FORCE ON THE STONE!
HOW MUCH FORCE CAN I EXERT?
HOWEVER MUCH THE RESISTANCE IS!
35

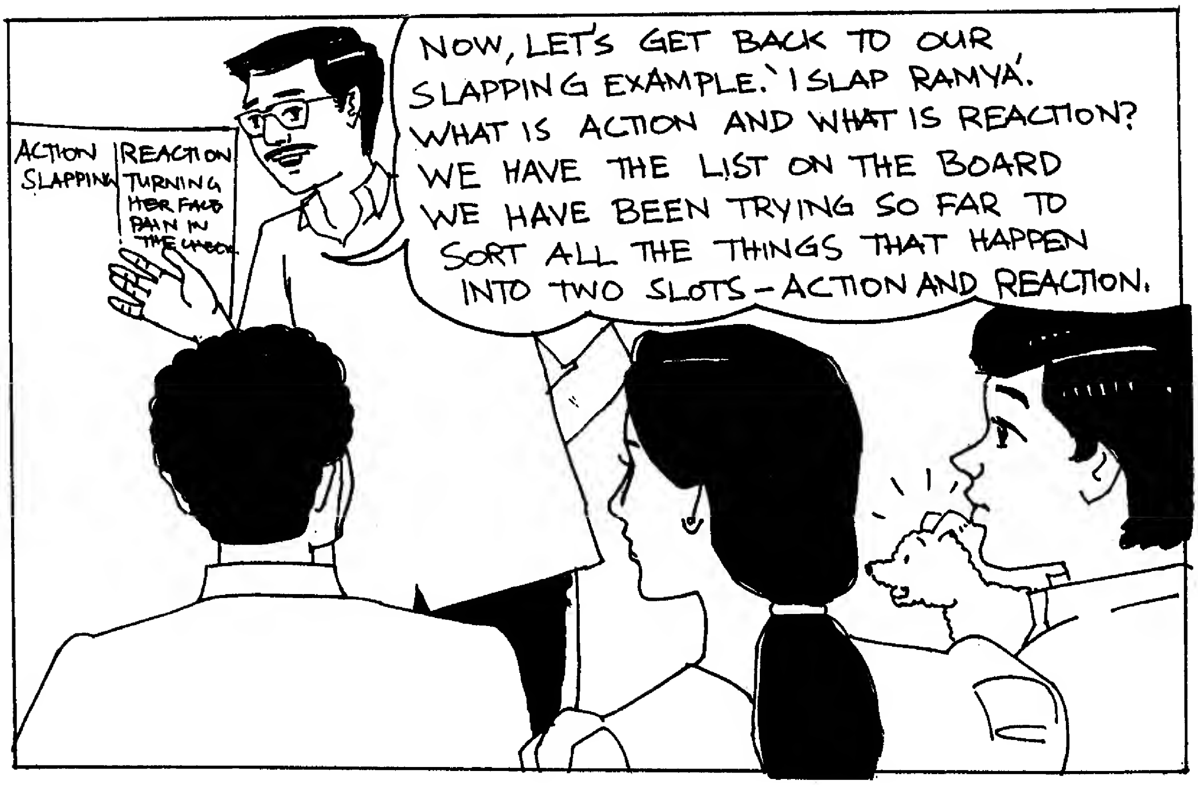
Exactly! That is what Newton's Third Law says. I can exert a force on you only because you resist me - you are trying to stop me or are refusing to move easily. Your resistance is again just another force - a force you exert on me. It is important to note that your resistance is what allows me to exert my force in the first place. Your resistance force is always equal to my force. My force is on you and your force (resistance) is on me. Resistance (or Reaction) **is not because of** Action. Action **does not cause** Reaction. If there was no Reaction (Resistance), I could not have exerted any Action (Force). So if anything, you should think of Reaction as allowing Action and not think of Action as causing Reaction. The fact is the neither Action nor Reaction is possible without the other. They both happen together - at the same time. If one increases, so does the other. Neither is the result of the other. There is no cause and effect relationship between Action and Reaction.

Please understand that we have NOT PROVED the third law. Our aim here is just to make sense of this law - to show that it really talks about what happens in the world around us. There is no proof for the third law. It is just a generalized statement made on the basis of a lot of observations of things around us.

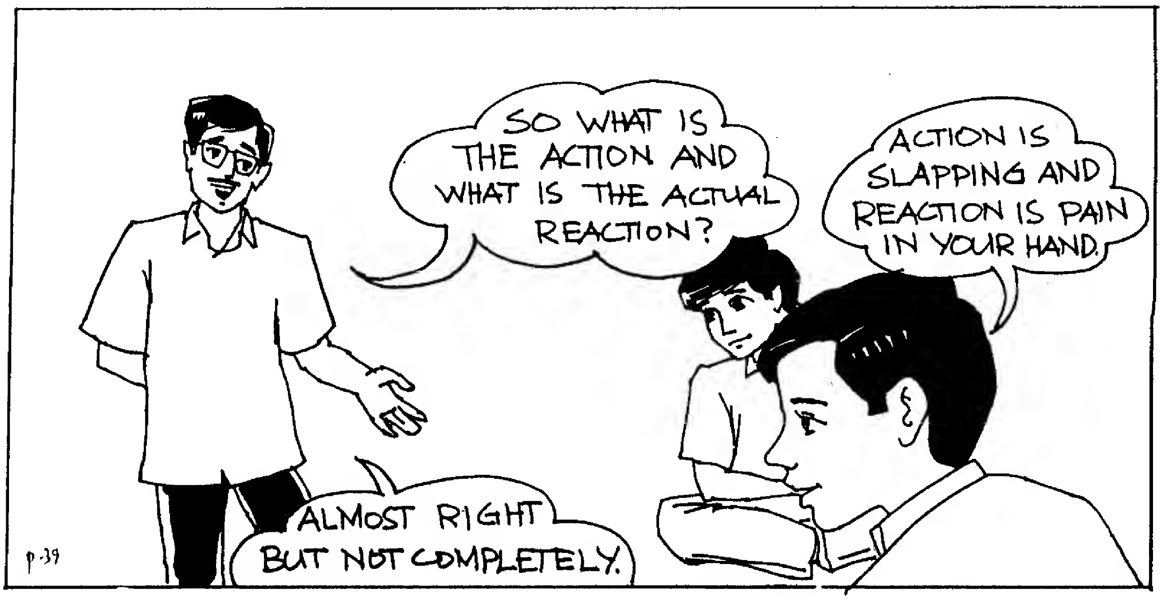


Summary so far

- ⇨**In Daily Language** Action and Reaction refer to things happening. Reaction is something that happens because of some Action.
- ⇨**In Physics,** by Action and Reaction we just mean Forces, not 'Doing'.
- ⇨Reaction **does not happen because of** Action. It happens **with** Action.
- ⇨**Newton's Third Law Says** - My Force on You is possible only because you resist and you resist by exerting a force on me. If you did not try to resist or stop me, I could not have exerted any force on you. At every instant of time, the force I exert on you is always equal to the force with which you resist me.




THERE IS A STRUCTURAL PROBLEM WITH THIS. WE SHOULD USE THREE SLOTS, NOT TWO!
1. ACTION WHICH IS A FORCE.
2. REACTION WHICH IS AGAIN ANOTHER FORCE BY WHICH THE OBJECT RESISTS.
3. RESPONSES TO THE ACTION OR REACTION.
RESPONSE IS THE THING THAT HAPPENS BECAUSE OF ACTION (OR REACTION).
THINK IN TERMS OF THESE THREE SLOTS AND TELL ME, IS CRYING A REACTION?
NO, IT IS A RESPONSE
WHAT ABOUT TURNING HER FACE?
AGAIN A RESPONSE!
WHAT ABOUT SLAPPING BACK?
IT IS ANOTHER ACTION AS A RESPONSE TO THE PAIN, WHICH IS A RES-PONSE TO THE SLAP.
THAT'S RIGHT. RESPONSES CAN ALSO BE ACTIONS (FORCES). RESPONSE TO A RESPONSE IS STILL A RESPONSE WE CAN CLUB IT WITH RESPONSES. ALL THE THINGS WE THOUGHT OF AS REACTIONS EARLIER ARE ACTUALLY RESPONSES. WHAT DOES NEWTON'S THIRD LAW SAY ABOUT ACTION AND RESPONSE? ABSOLUTELY NOTHING - NOT A WORD. THE RESPONSE CAN BE ANYTHING AND EVERYTHING. ALL NEWTON SAYS IS THE REACTION (OR RESISTANCE) IS EQUAL TO THE FORCE EXERTED ON RAMYA'S CHEEK.



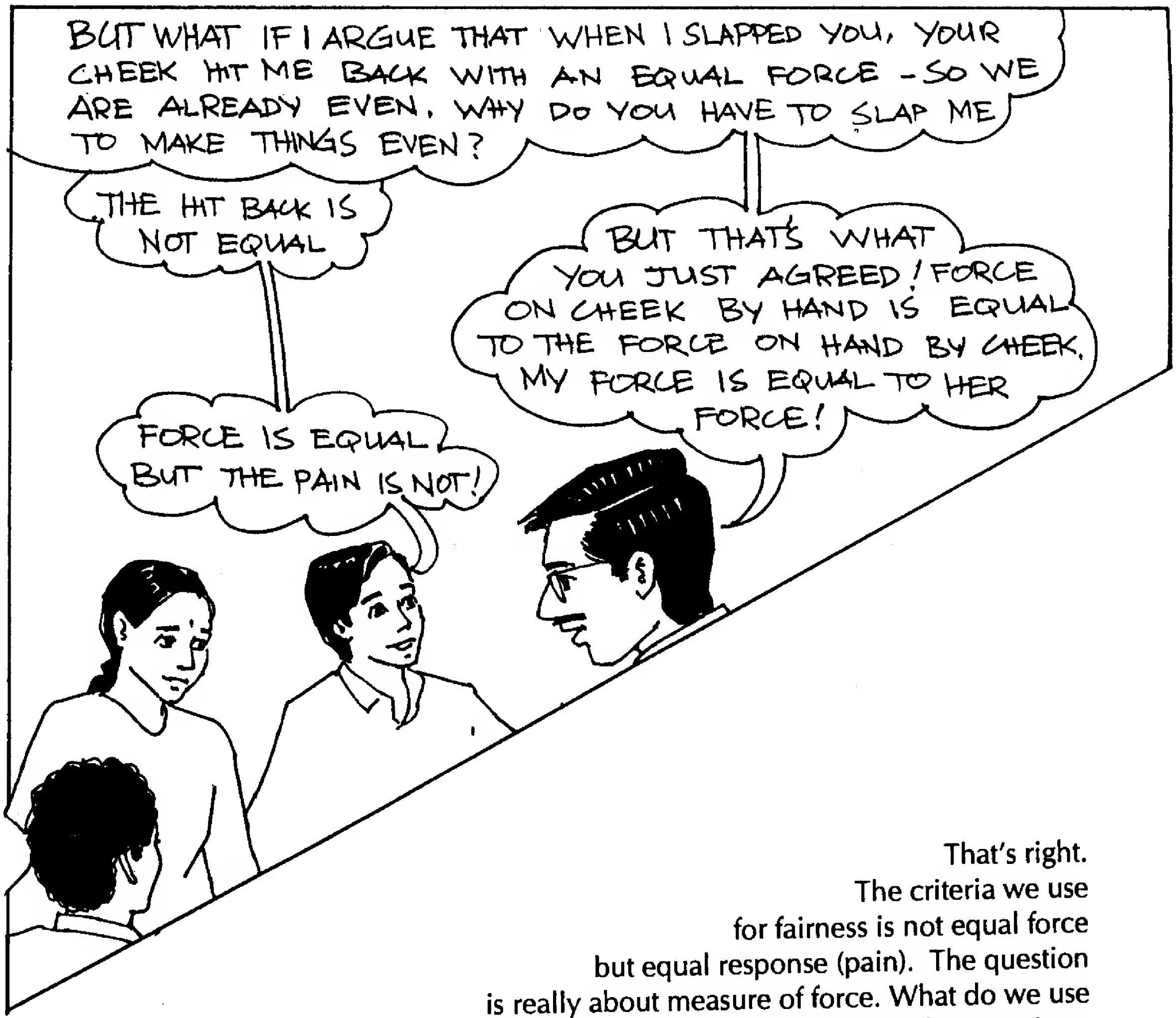
In physics Slapping is not "action". In ordinary language doing is action and slapping is doing. But in physics action means force - not the doing. The action we want in physics is "the **force my hand exerts** on Ramya's cheek when I slap".

Action is not the slapping itself - it is only the force on her cheek by my hand. Similarly the reaction is the force her cheek exerts on my hand. My hand's pain is a response to this force - but it is not the reaction. Reaction is again just a force.

Remember !

Force always come with prefixes and suffixes. Don't ever talk of "the force" all by itself. Always say "Force on A by B". Force on Ramya's cheek by my hand. The reaction is always the interchange of B and A. Force on my hand by Ramya's cheek. These two are always equal, always on different bodies and always opposite. That's Newton's third law. So the only reaction in this case is the force Ramya exerts on my hand and that is equal and opposite to my force on Ramya's cheek.


ACTION-REACTION-RESPONSE
SLAPPING-TURNING FACE
PAIN IN CHEEK
PAIN IN HAND
SOUND
CRYING
HAND MARKS ON FACE
SLAPPING BACK
EMOTIONS
ALL THE ABOVE ARE RESPONSES NOT REACTIONS
THIS IS WRONG! ACTION IS FORCE BY MY HAND ON HER CHEEK
THE ONLY REACTION IS THE FORCE BY RAMYA'S CHEEK ON MY HAND.
IS IT ALL CLEAR NOW?
YES!
IN THAT CASE ANSWER THIS: I SLAP YOU AND YOU SLAP ME - WILL YOU THINK OF THIS AS FAIR?
NO. SHE SHOULD SHOW THE OTHER CHEEK.
WE WILL LEAVE OUT JESUS FOR THIS CLASS. TELL ME IN REALITY, WHETHER YOU WILL THINK OF HER SLAPPING ME AS FAIR?
YES!



That's right. The criteria we use for fairness is not equal force but equal response (pain). The question is really about measure of force. What do we use to measure the quantity of force? Often people use pain or distance moved (as on the carom board) as the measure of how much force one has exerted. This can be tricky. Pain or response to a force is not always a good measure of force. In this example, pain felt by my hand is much less than the pain felt by your cheek. But the force exerted by my hand on your cheek is equal to the force exerted by your cheek on my hand. Same force on different objects can produce different responses. But reaction to the same force is always the same. Now the question is if pain is not a good measure of force, what is? This is a hard question. We can only measure force by measuring the response to it. But to get a fair quantification, different forces must act on the **same** body to produce different responses. When forces act on different bodies and produce different responses - one cannot be sure whether the difference in response is caused by difference in forces or difference in bodies.

Chapter 1: Section C

Tug of War

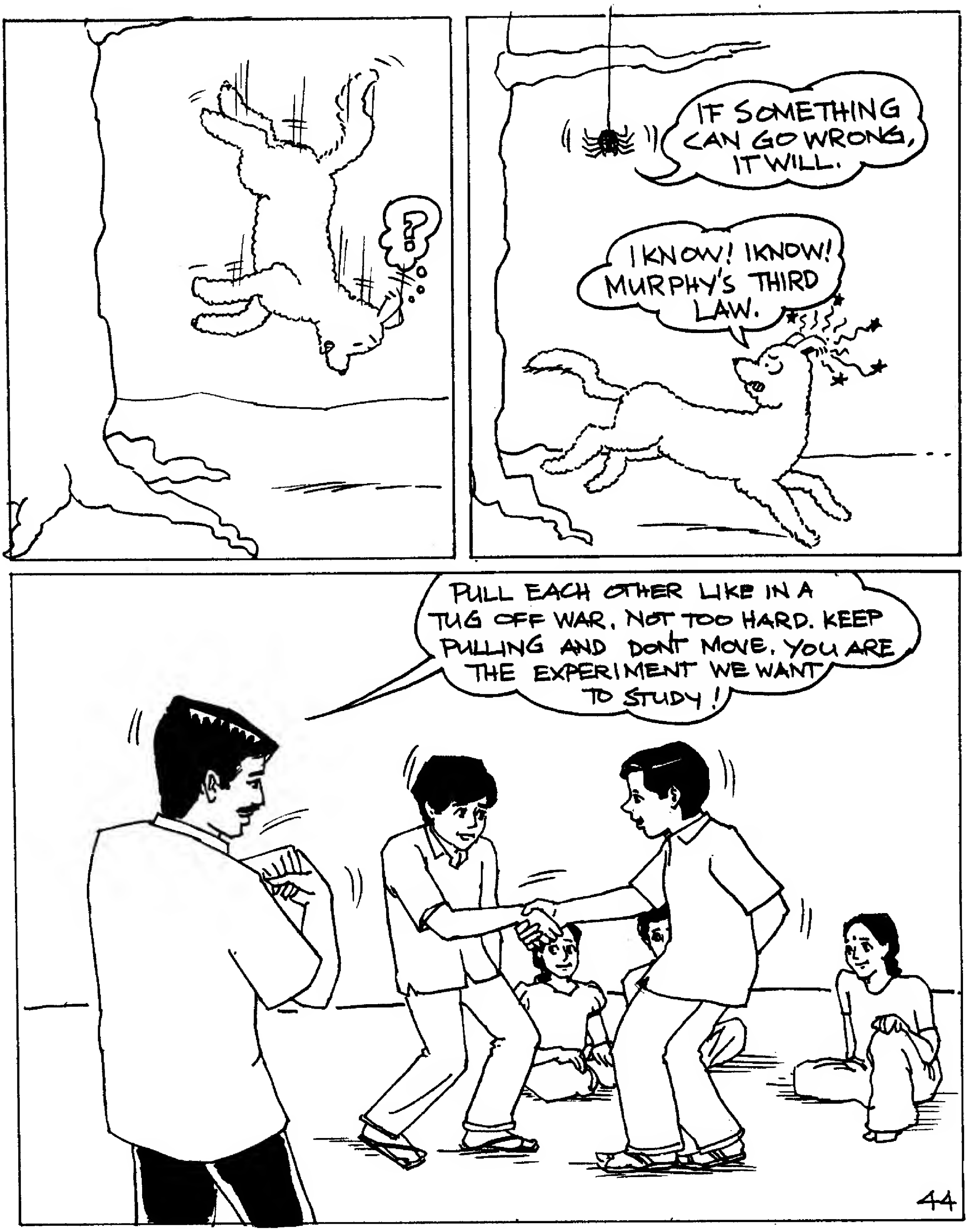
In the previous section we saw what Newton's third law really says. In this section, we will apply it to a tug of war situation.

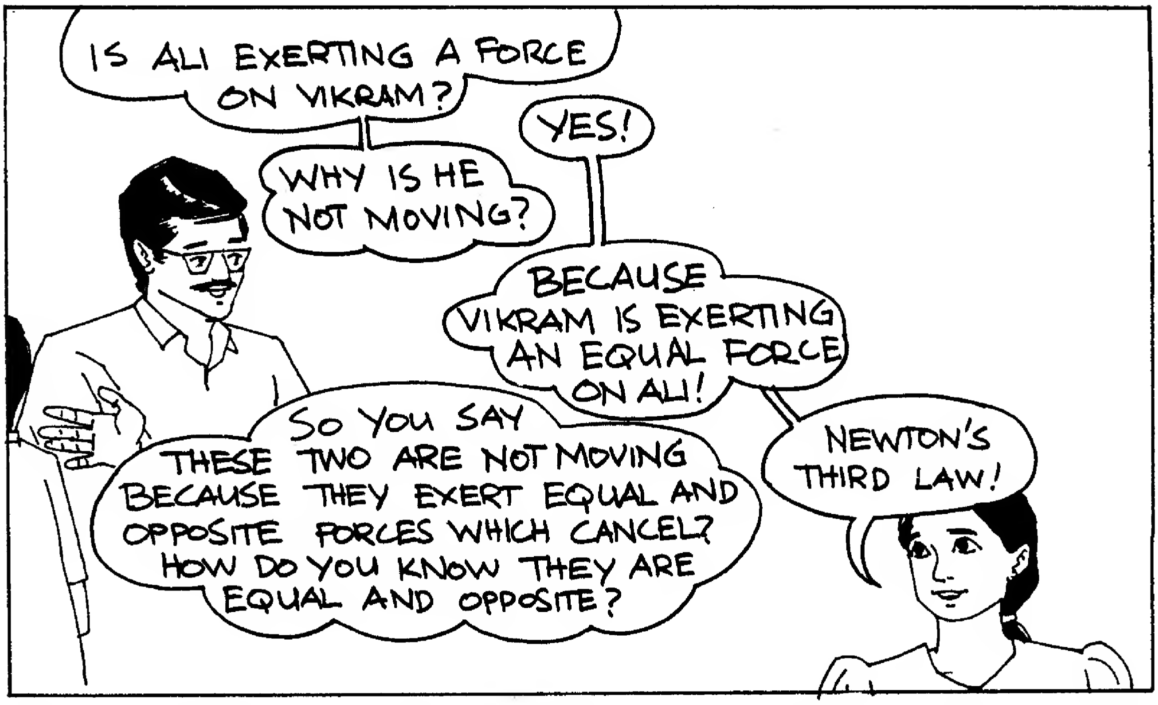
Often students believe that since action and reaction are always equal and opposite, they must always cancel. It is easy to say that Action and Reaction are on different bodies and therefore cannot cancel. But just saying this doesn't help students internalize this idea. In this section, we look at a number of examples to overcome this mental block. When an object is not moving, we know that somehow the net force on the object must be zero. The tricky job is finding how. To do this one needs to find all the forces on the object and then see how they together cancel. When I pull you, and you don't move - two forces on you must be canceling. We show that they cannot be my pull on you and your pull on me. What then are the forces that do cancel? That's what we will try to answer in this section.

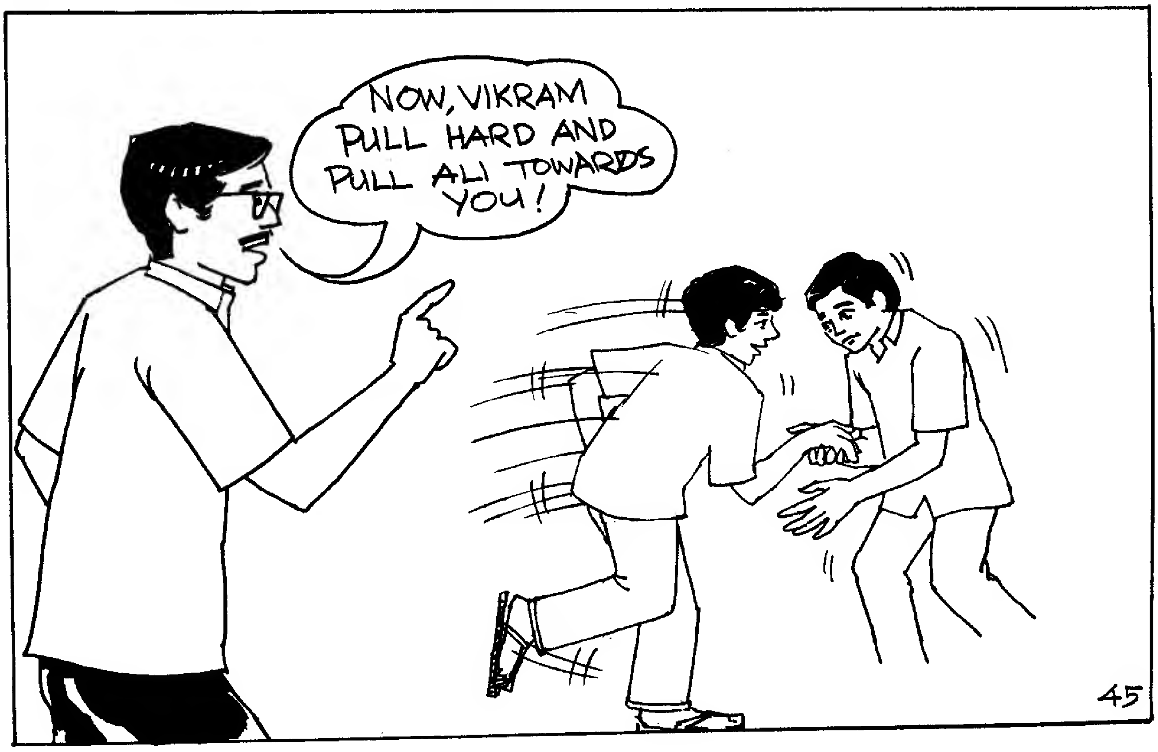
Starting with this simple example, we move on to understanding the framework for Newton's Laws. We look at Newton's Zeroth, Halfth and Three-fourth Laws. Then we come back to analyze the tug of war situation.

This section is the hardest to follow - but it is also the most important section in this chapter. By the end of this section the reader can identify the specific forces in a tug of war. The reader will also have a clearer understanding of Newton's third law and particularly its validity even when things are moving. She will also understand why Action and Reaction can never cancel. Finally the reader will also get a glimpse of the overall Newtonian framework within which all these laws make sense.

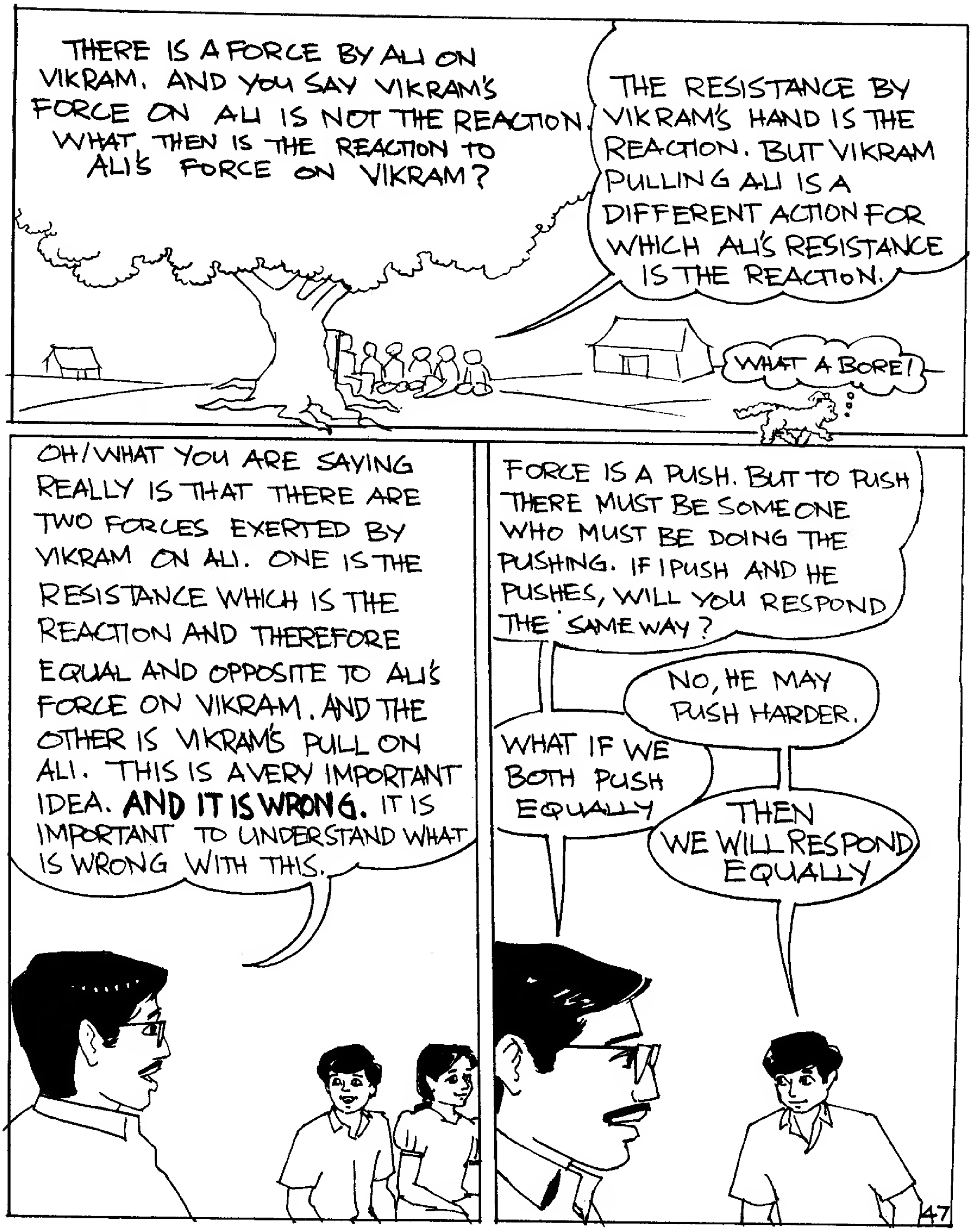

OK. NOW LET'S TRY A NEW EXPERIMENT. ALI AND VIJAY COME HERE!
OOPS... I'M SORRY!
ALL NICE TO SAY SORRY... WHY DOESN'T HE LOOK WHERE HE IS GOING?
AT ANY RATE I WILL BE SAFE UP HERE!
PEACE AT LAST!
WOOOAH!

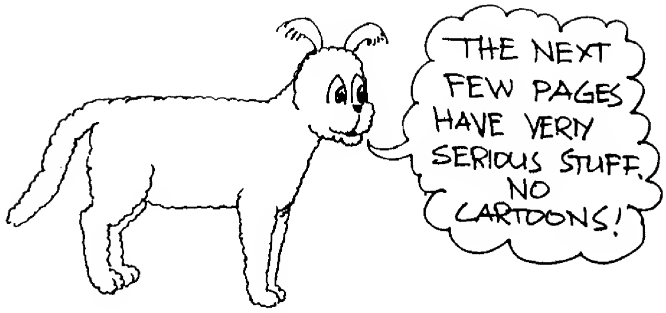

?
IF SOMETHING CAN GO WRONG, IT WILL.
I KNOW! I KNOW! MURPHY'S THIRD LAW.
PULL EACH OTHER LIKE IN A TUG OFF WAR. NOT TOO HARD. KEEP PULLING AND DON'T MOVE. YOU ARE THE EXPERIMENT WE WANT TO STUDY!


IS ALI EXERTING A FORCE ON VIKRAM?
WHY IS HE NOT MOVING?
YES!
BECAUSE VIKRAM IS EXERTING AN EQUAL FORCE ON ALI!
NEWTON'S THIRD LAW!
SO YOU SAY THESE TWO ARE NOT MOVING BECAUSE THEY EXERT EQUAL AND OPPOSITE FORCES WHICH CANCEL? HOW DO YOU KNOW THEY ARE EQUAL AND OPPOSITE?


NOW, VIKRAM PULL HARD AND PULL ALI TOWARDS YOU!

IF THE FORCES BY ALI AND VIKRAM ON EACH OTHER ARE EQUAL, WHY DID ALI MOVE?
NOW THEY ARE NOT EQUAL!
BUT IF THOSE TWO FORCES ARE ACTION-REACTION THEY SHOULD ALWAYS BE EQUAL, ISN'T IT?
HOW CONFUSING! WHY DOESN'T HE EVER COME TO THE POINT?
ARE THE TWO FORCES - THE FORCE BY ALI ON VIKRAM AND FORCE BY VIKRAM ON ALI - AN ACTION -REACTION PAIR?
YES!
NO
HOW MANY SAY "NO THEY ARE NOT ACTION REACTION PAIRS?"


THERE IS A FORCE BY ALI ON VIKRAM. AND YOU SAY VIKRAM'S FORCE ON ALI IS NOT THE REACTION. WHAT THEN IS THE REACTION TO ALI'S FORCE ON VIKRAM?
THE RESISTANCE BY VIKRAM'S HAND IS THE REACTION. BUT VIKRAM PULLING ALI IS A DIFFERENT ACTION FOR WHICH ALI'S RESISTANCE IS THE REACTION.
WHAT A BORE!
OH! WHAT YOU ARE SAYING REALLY IS THAT THERE ARE TWO FORCES EXERTED BY VIKRAM ON ALI. ONE IS THE RESISTANCE WHICH IS THE REACTION AND THEREFORE EQUAL AND OPPOSITE TO ALI'S FORCE ON VIKRAM. AND THE OTHER IS VIKRAM'S PULL ON ALI. THIS IS A VERY IMPORTANT IDEA. AND IT IS WRONG. IT IS IMPORTANT TO UNDERSTAND WHAT IS WRONG WITH THIS.
FORCE IS A PUSH. BUT TO PUSH THERE MUST BE SOME ONE WHO MUST BE DOING THE PUSHING. IF I PUSH AND HE PUSHES, WILL YOU RESPOND THE SAME WAY?
NO, HE MAY PUSH HARDER.
WHAT IF WE BOTH PUSH EQUALLY
THEN WE WILL RESPOND EQUALLY



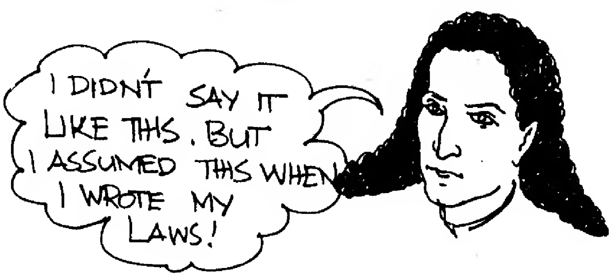
We assume that an object responds equally to equal force. This is a very fundamental assumption in physics - that how you respond depends only on the force on you. This is not at all obvious. If your friend slaps you and your mother slaps you - even if they slap with the same force you respond differently. We respond differently to the same action by different people. So this assumption is not at all an obvious assumption to make. But we still make it and it is very fundamental to all of our physics. Let's spend a short while on this point...

Why do we respond differently to different people slapping ? Apart from the force of the slap - there is also the knowledge of who slapped. Suppose you are blind folded and you don't know - even then you can respond differently to the same slap depending on your mood. But then we can say your mood is changing you. After a number of such refinements, we finally come to a stage where you don't change, don't have any other knowledge. Then we assume that you will respond the same way to the same force. This makes sense easily for in-animate things, but we also assume this true for living beings.

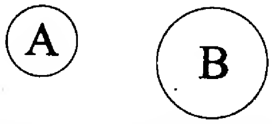
Why is this so fundamental an assumption? Because without this - Force has no meaning. Every time I say there is a force on you, I have to worry about whether this force was because of me, because of your mother, the earth, moon etc. What you will do will depend not only on the force but also on who gave you the force. If all the time I have to remember what caused the force, then what is the point of 'force'? We might as well talk of mother pushing, earth pulling etc. The following is very critical for physics - so fundamental that I will call it Newton's Zeroth Law. It comes before all other laws.

Newton's Zeroth Law

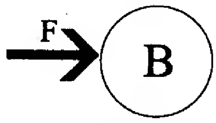
Every particle which acts or disturbs another particle does it through a vector quantity called Force. The entire action of the first particle on the second is **completely** determined by the force it exerts on the second. We can **replace** the first particle by its force.



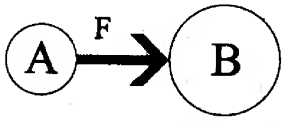
It is important to understand what this means. Let's take two objects A and B. A disturbs B or acts on it. How will B respond ? This is entirely determined by A's force on B. Once I put this force on B, A can be removed from the scene - there is no more A as far as B is concerned. This means B can see the situation in terms of either of the two pictures below...



B says A is doing something to me



B says there is no A, just a Force on me
This force is from A, but once the force is put there, A plays no role.



This picture B cannot see.
Either there is A's force or there is A - there can't be both.
Note - A is being **replaced** by its force.

Physicists often use pictures like this - with both A and F - but that's because they are lazy and don't want to draw two pictures. But the truth is it is wrong in principle. F arises because we want to replace A with F.

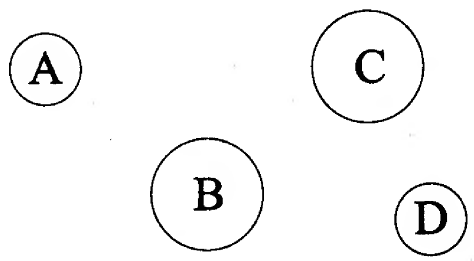
This is critical to all Newton's laws - an object exerts one force and that's it. It can do nothing more. Why are we stuck to the earth ? We can either say "The earth is pulling me down" - or we can say "I have a force on me" - Both statements are equivalent. Both are ***alternate*** descriptions - but you cannot use both at the same time.

T: Exactly - that's what we assume. Again in human terms if I am fighting and I see one person, I will fight. With two people, I may fight. But if there are three of you - I will run away. Here the two people's actions on me are not independent of each other. But this is the assumption we will make in our physics. The idea is that 'force' exerted by a body is independent of other objects nearby. You have to think very hard about this. It is not at all obvious. The assumption we make here is another important law... what I will call the 'half-th' law...

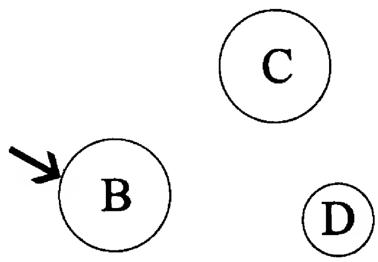
Newton's Half-th Law...

Each particle acting on another particle does so **independently** of other particles. This means the force exerted by a particle on another does not depend on what other particles nearby are doing. The net force on a particle is the resultant of all the forces on the particle by all the other particles.

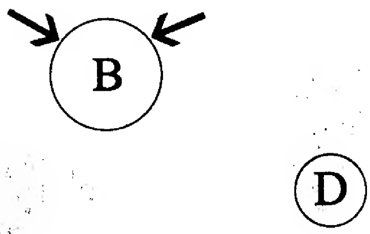
This means if A exerts a force on B - this force may depend on what B is doing, how B looks, how far B is, etc and similarly on properties of A. But this force won't change just because C has come near it. This is very critical even in the zeroth law. If A's action on B changes because of C, then I can't really replace A by its force. If the half-th law were not true, then I can only jointly replace all objects by one force. This would be very messy. Luckily, the half-th law works. So we can replace each object by its force and then based on all these forces find out what happens to B.



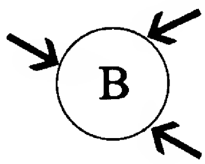
A, C, D all act on B



A is replaced by its force on B



Then C is replaced by its force on B

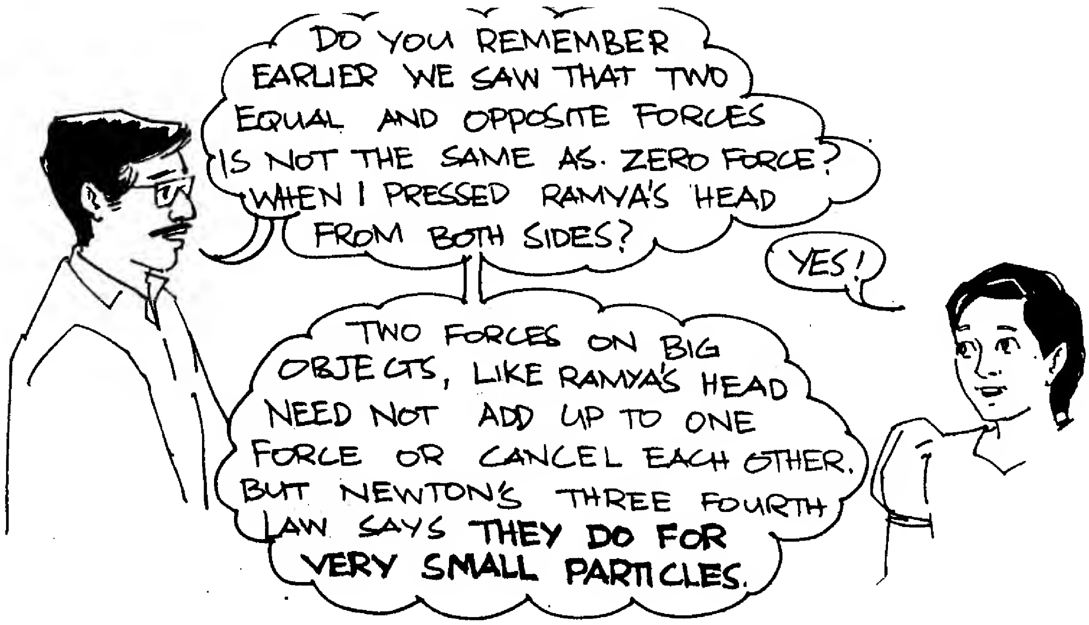


D is also replaced.
B response depends only
on these three forces

The above step-by-step process is possible only because the each particle acts independently. This does not mean the response of B does not depend on C and D - it does. But the Action (force) on B because of A does not change. B responds not just to this force but rather to the resultant of all the forces on it.

This is a bit tricky to understand - you will have to think very carefully about this to understand what this means. B's response changes because of other fellows around - but A's action or force on B does not. This is what allows us to get rid of each object one by one and replace by its force.

There is one more law - Newton's three-fourth law. I promise that is the last law before we go on to our usual first, second and third laws! This law tells us what to do when we have many forces.



Newton's three-fourth Law!

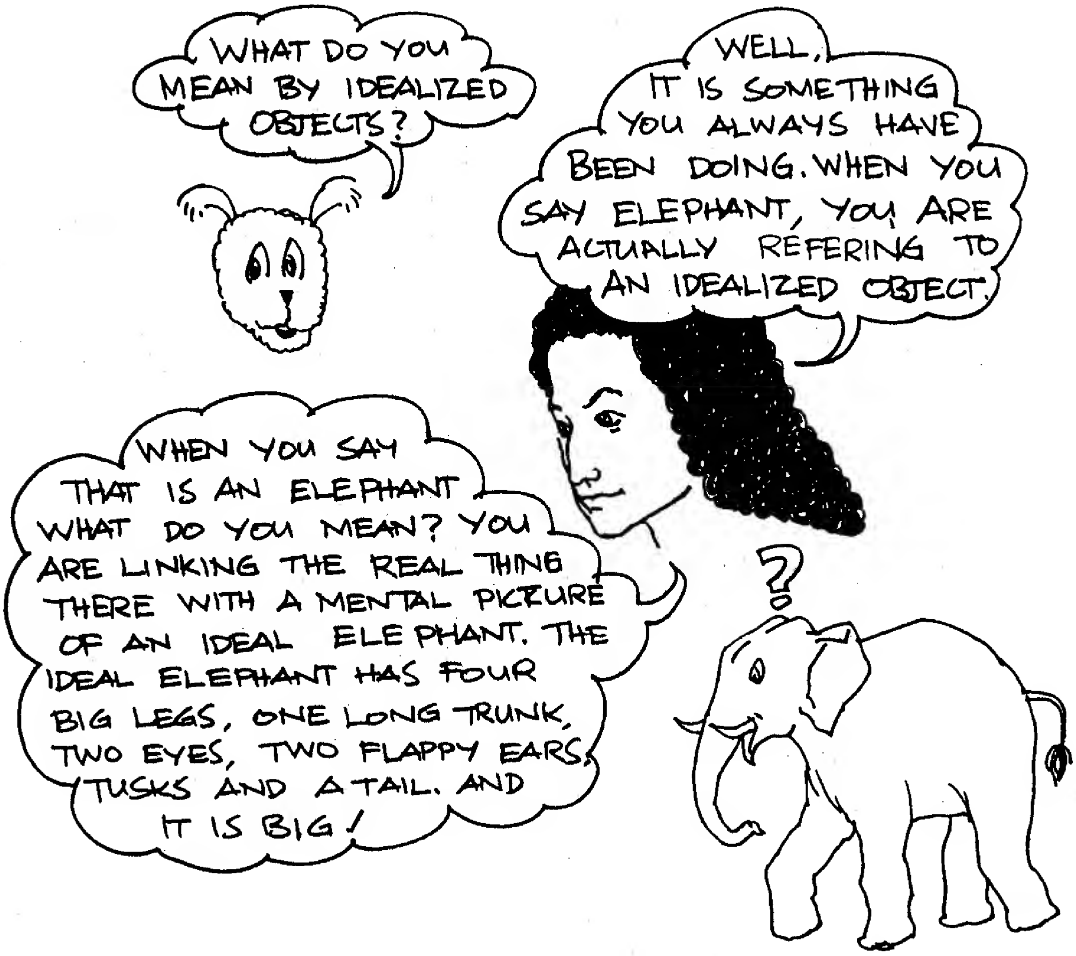
If there are several forces on a particle, it behaves as if there is only one net force - this net force can be found by a special addition called vector addition of forces. We can add forces by placing arrows tail to head.

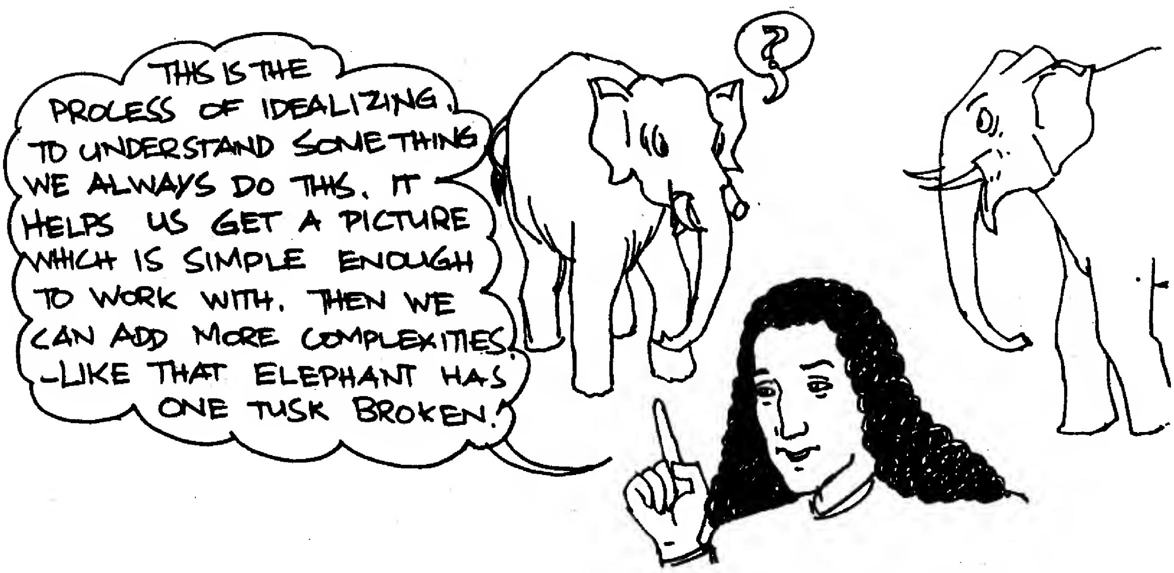
Many times we can think of big objects which cannot feel pain, twist, change shape, compress etc as particles - in that case this three-fourth law will apply to those objects as well.

Now that we know these three laws, let's quickly go over the basis for the Newton's framework for mechanics.

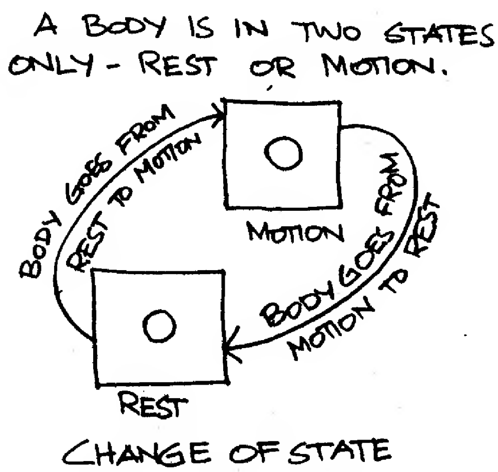
Newton wanted to study motion of objects - quantitatively. But the real world is very complex - too many confusing things. When you take real objects and begin to quantify, there are so many factors that it becomes even difficult to start the quantification process.

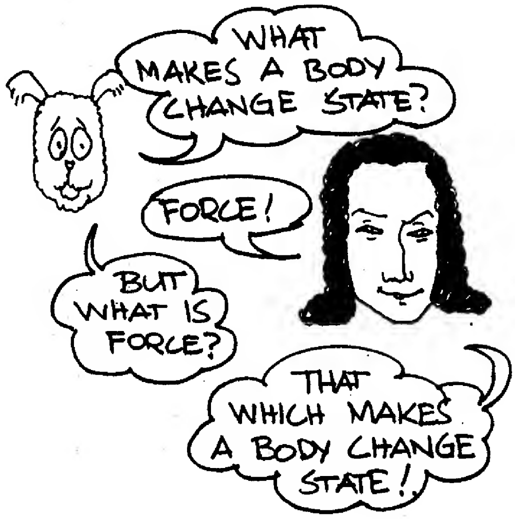
So Newton came up with an idea. He selected a few relevant factors and created ***idealized*** objects. Whether it is an ant or an elephant - he decided to call it 'body'.





Newton's aim was studying motion. So his ideal body could only do two things - be at rest or be in motion. Real bodies do other things - they shake, they are of varying sizes, they may never be at rest. But all this was not fundamental as far as Newton was concerned. In his ideal world, they did not exist. Now the point is not whether Newton make such simplifications - the real point is whether by doing all this, he can predict the motion of real objects around us.






THAT'S NOT FAIR! YOU CAN'T GIVE CIRCULAR ANSWERS!
AND YOU MUST UNDERSTAND THAT WE ARE CONSTRUCTING A MODEL. BY DEFINING IDEAL OBJECTS AND INVENTING CONCEPTS LIKE FORCE, I AM GOING TO SEE IF I CAN FIND SOME LAWS AND RELATIONSHIPS. MY HOPE IS THAT WITH THESE LAWS I CAN PREDICT WHAT IDEALIZED BODIES DO!
BUT I WANT TO KNOW WHAT HAPPENS TO REAL BODIES. NOT TO SOME STUPID IDEALISED BODIES!
YEAH!
WELL! THAT'S WHAT I AM HOPING TO DO AS WELL. THE MOTION OF THESE IDEALIZED BODIES I AM HOPING WILL ALSO PREDICT ACCURATELY WHAT REAL BODIES DO!
BUT HOW ARE YOU SURE YOUR PREDICTION WILL WORK FOR US? WE DON'T LOOK LIKE YOUR IDEALIZATION.
I AM NOT SURE. ONE HAS TO PREDICT AND SEE. OVER THE YEARS PEOPLE HAVE EXPERIMENTALLY TESTED THIS MODEL TO SEE IF IT WORKS WITH REAL OBJECTS.

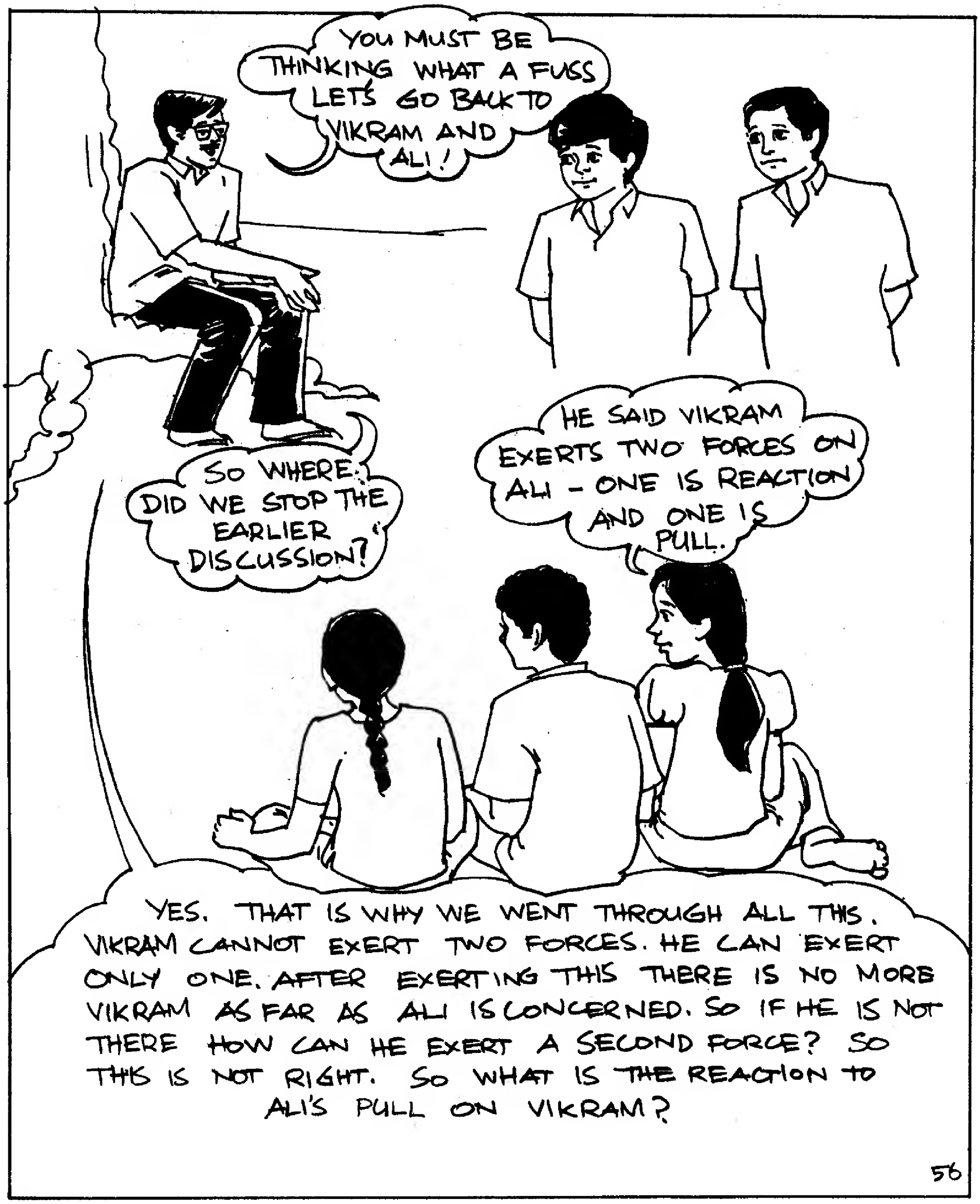
One has to understand this process of model building. We do this all the time in science. Reality is very difficult to study directly - too many parameters and factors to worry about. So we construct a simplified model. In this model we choose what are the relevant factors and what are not. This way we can deal with just a few factors and play around with these, make up laws and try to predict what will happen.

Our model may be a bad one. What is predicted for the ideal case may be very far from what really happens. Then we dump the model and build a new one. On the other hand our model may work very well - then we keep it and use it to predict the real world.

Newton created such a model for the real world. In his model, bodies have one significant character called mass. (The rest of the stuff is added in later - but they are less important than this "mass"). Bodies are in two states - either at rest or are moving. In Newton's model forces do the job of changing the state of a body. (Bodies may be in different motion states - but this is a just straight forward extension, so we will not worry about that now). What exactly is force is not very clear - for all practical purpose it is that which changes the state of the object. To make a moving object come to rest or to get an object at rest moving, you need force.

In addition Newton's model assumes a number of things about this force - these are what we stated in the zeroth, halfth and three-fourth laws. We have a lot of objects 'influencing' another object. Each of these objects can be removed and replaced by a corresponding force. What this force is depends on that influencing object and the influenced object. Other objects nearby are silent - they don't change this force. After we get all the forces, we would have replaced all the influencing objects. We are now left with just one object and forces on it. Then we can convert all these forces (if the object is like a particle) into one single force using a procedure called vector addition. This is the way in which all actual physical situations can be broken down into a case of one particle like object with one force on it.

All these ideas together with Newton's 3 laws of motion form the basis for Newton's model. We need to understand what this model is all about and then we need to use this model to predict what happens in different cases. If Newton's model is a good one, it should be able to explain the real world reasonably well. This is what Newtonian Mechanics aims to do.


YOU MUST BE THINKING WHAT A FUSS LETS GO BACK TO VIKRAM AND ALI!
SO WHERE DID WE STOP THE EARLIER DISCUSSION?
HE SAID VIKRAM EXERTS TWO FORCES ON ALI - ONE IS REACTION AND ONE IS PULL.
YES. THAT IS WHY WE WENT THROUGH ALL THIS. VIKRAM CANNOT EXERT TWO FORCES. HE CAN EXERT ONLY ONE. AFTER EXERTING THIS THERE IS NO MORE VIKRAM AS FAR AS ALI IS CONCERNED. SO IF HE IS NOT THERE HOW CAN HE EXERT A SECOND FORCE? SO THIS IS NOT RIGHT. SO WHAT IS THE REACTION TO ALI'S PULL ON VIKRAM?
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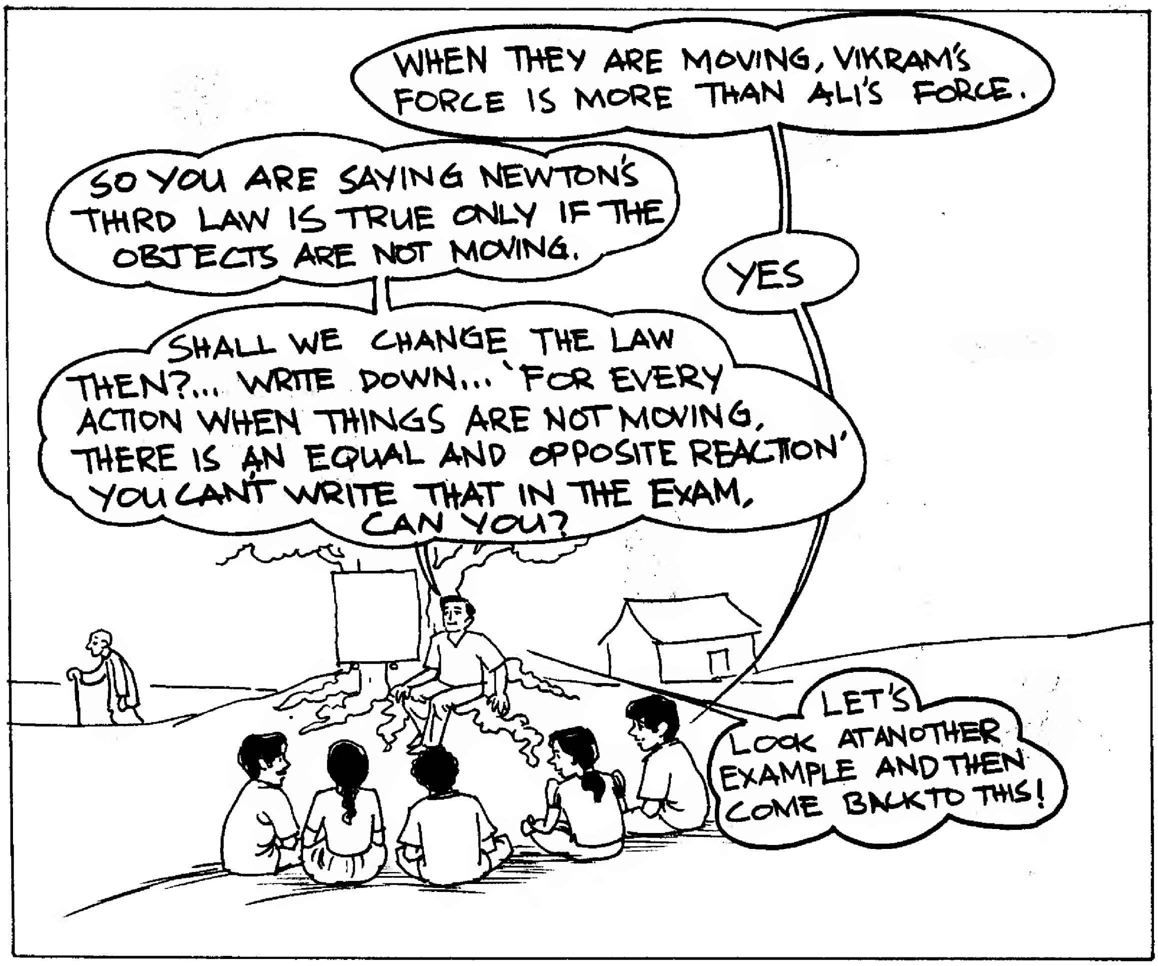
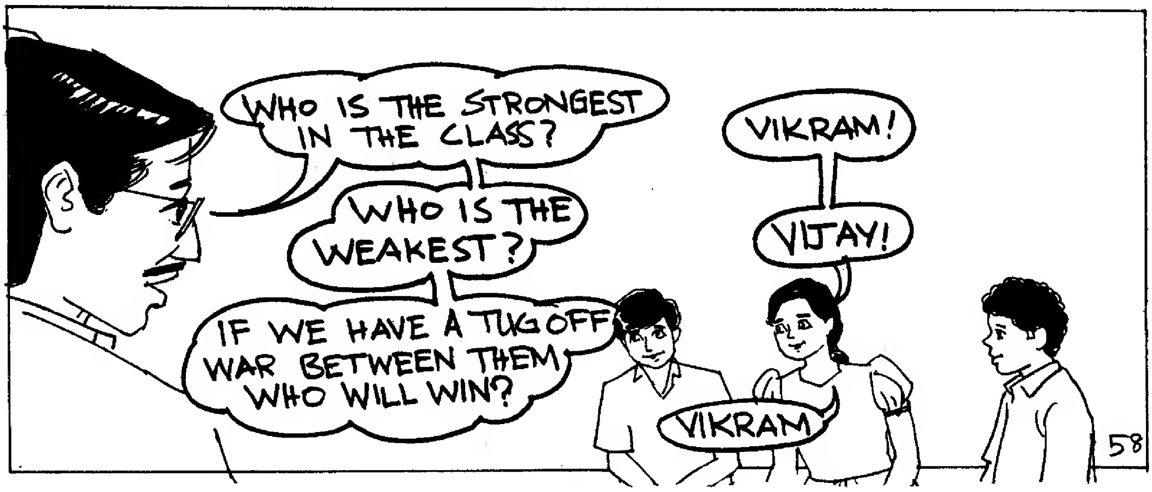
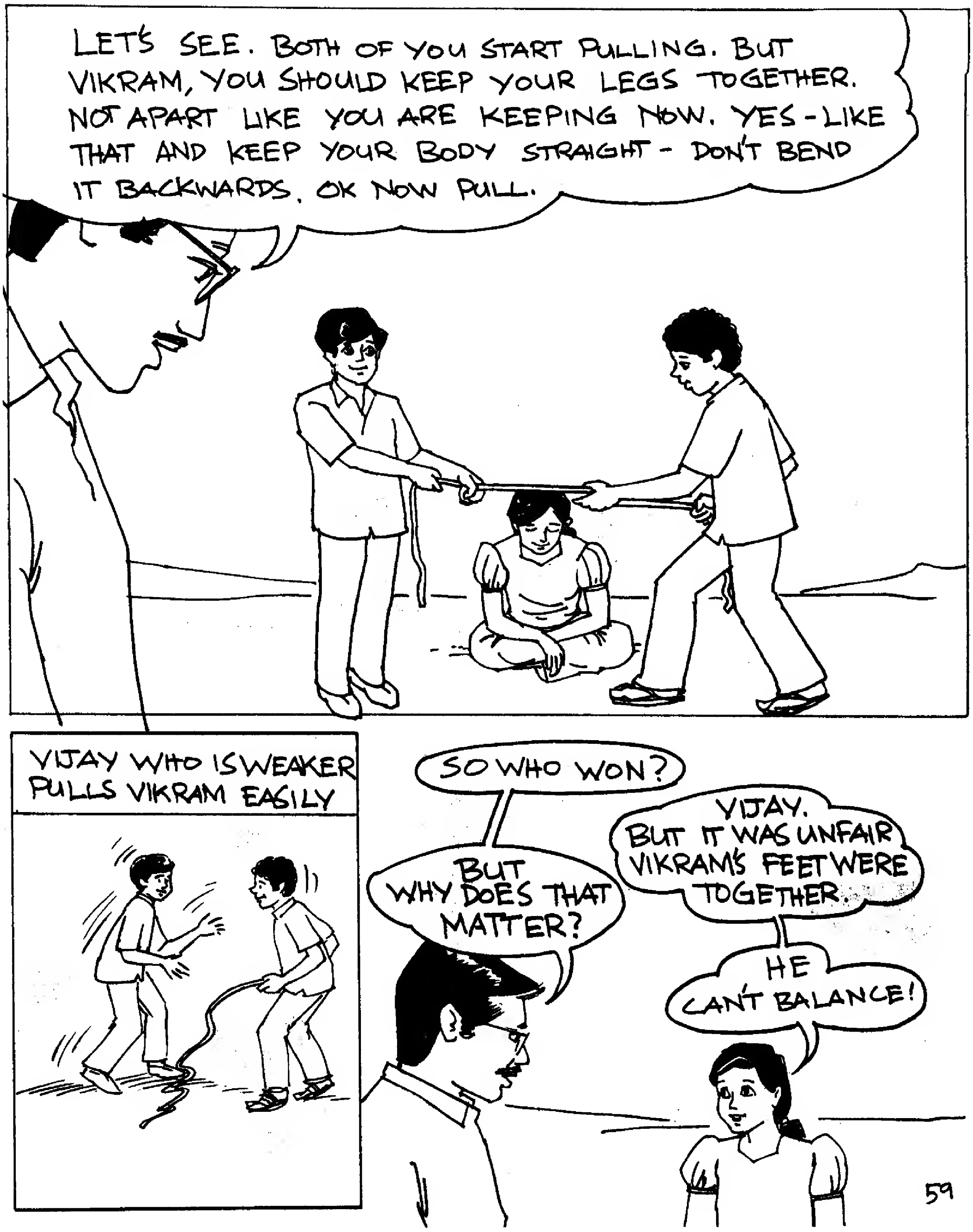
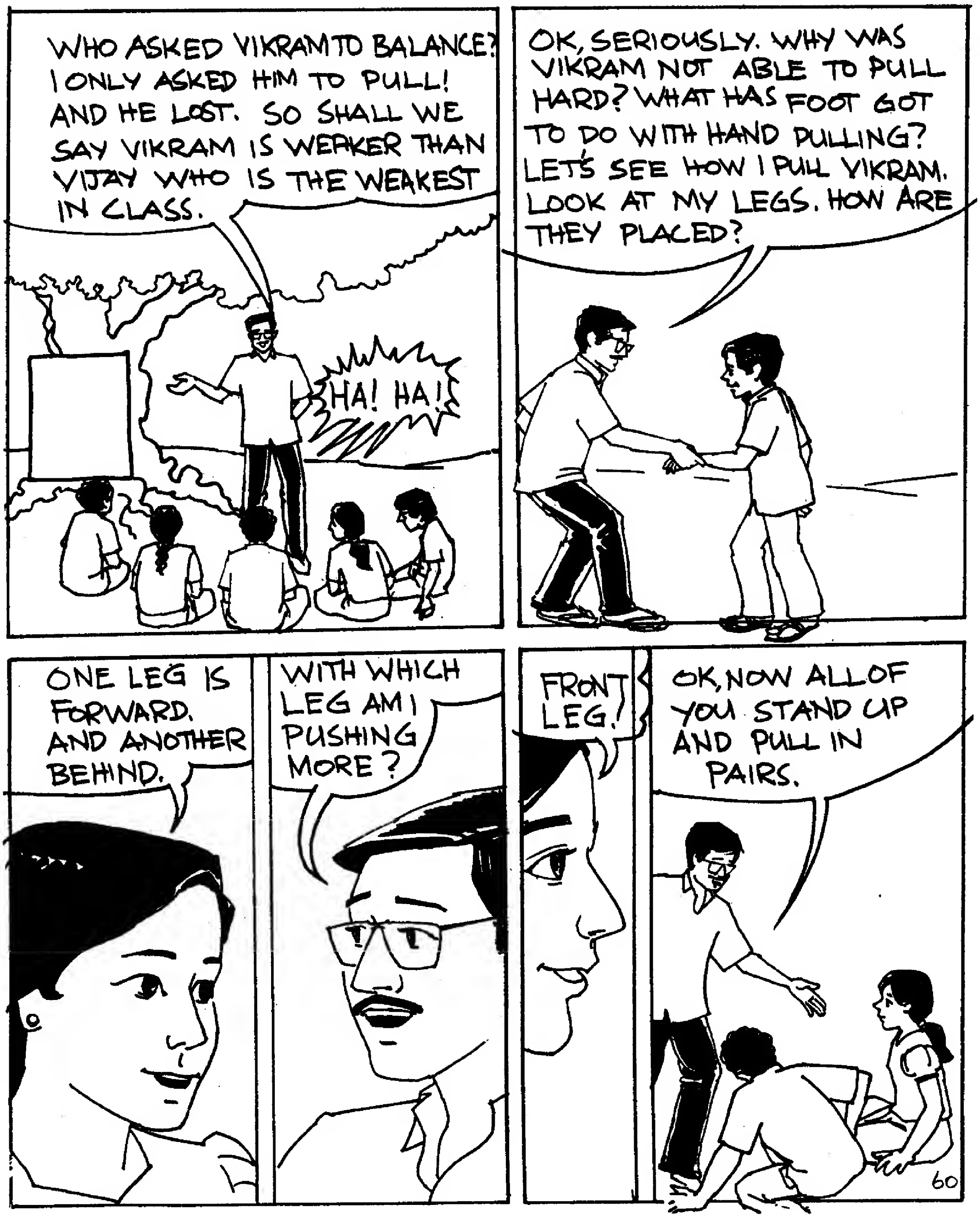
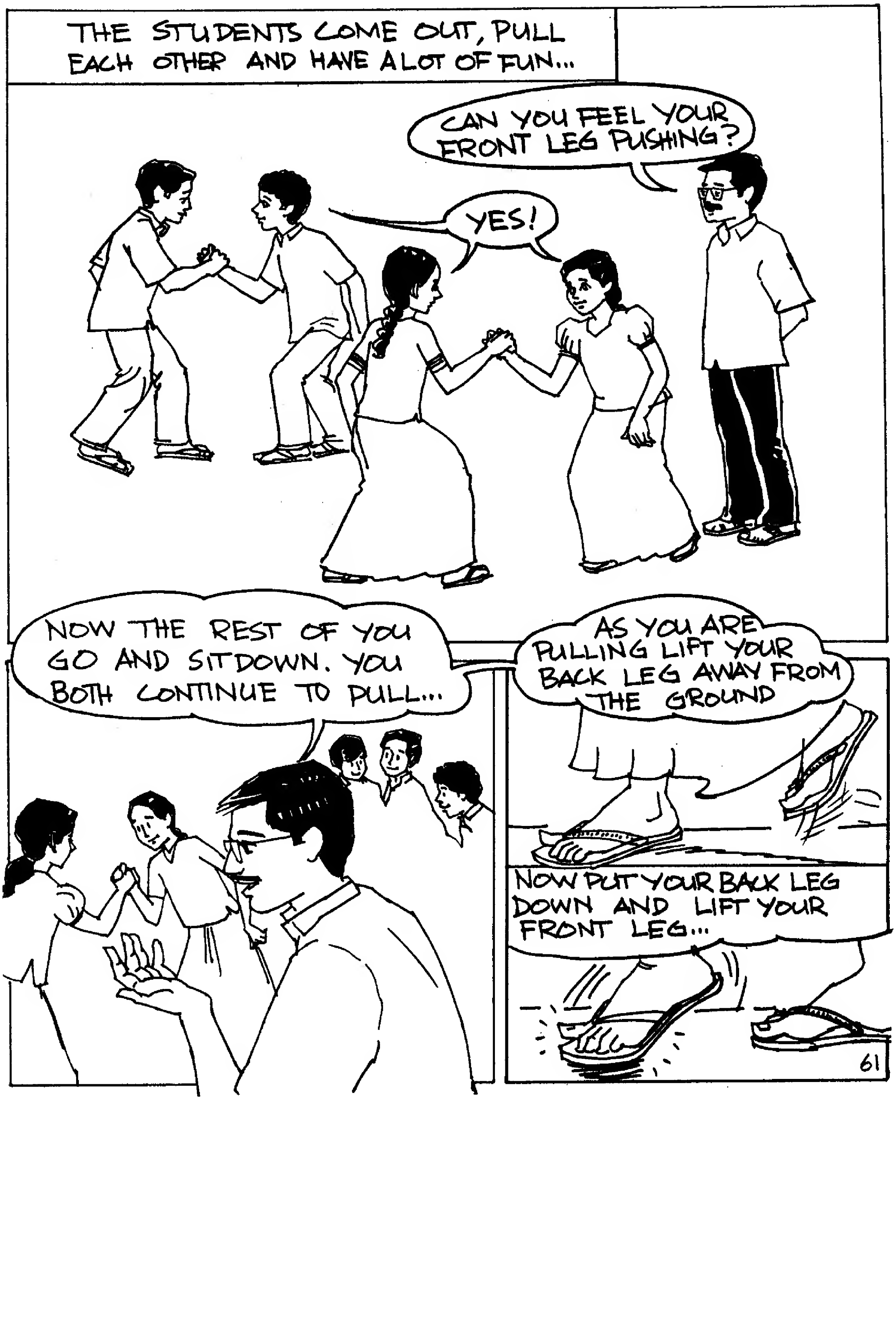
First - Vikram and Ali pulled and they did not move. ⇨ You said it was because their forces were equal and cancelled. ⇨ I asked why they were equal - you said Newton's third law. ⇨ Then Vikram pulled Ali towards him. ⇨ I asked if *again* the forces cancel - you all said no. ⇨ But Newton says for each Action there is equal Reaction. So what is the reaction to Vikram's pull? ⇨ You said - there are two forces from Ali - one is the reaction and the other is the pull. We saw that is not possible. There is only one force. So the only possible reaction seems to be Ali's pull. ⇨ Now - Newton says these two are equal. Then as you said earlier they should cancel always - which means Vikram can never pull Ali towards him. But he just did! ⇨ Something is wrong - what is it?

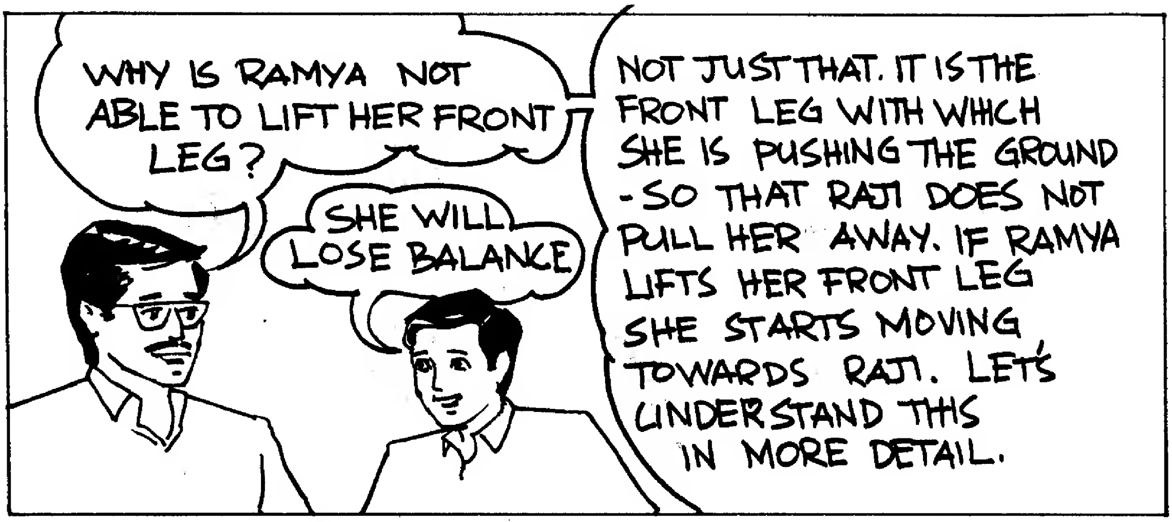

WHEN THEY ARE MOVING, VIKRAM'S FORCE IS MORE THAN ALI'S FORCE.
SO YOU ARE SAYING NEWTON'S THIRD LAW IS TRUE ONLY IF THE OBJECTS ARE NOT MOVING.
YES
SHALL WE CHANGE THE LAW THEN?... WRITE DOWN... 'FOR EVERY ACTION WHEN THINGS ARE NOT MOVING, THERE IS AN EQUAL AND OPPOSITE REACTION' YOU CAN'T WRITE THAT IN THE EXAM, CAN YOU?
LET'S LOOK AT ANOTHER EXAMPLE AND THEN COME BACK TO THIS!

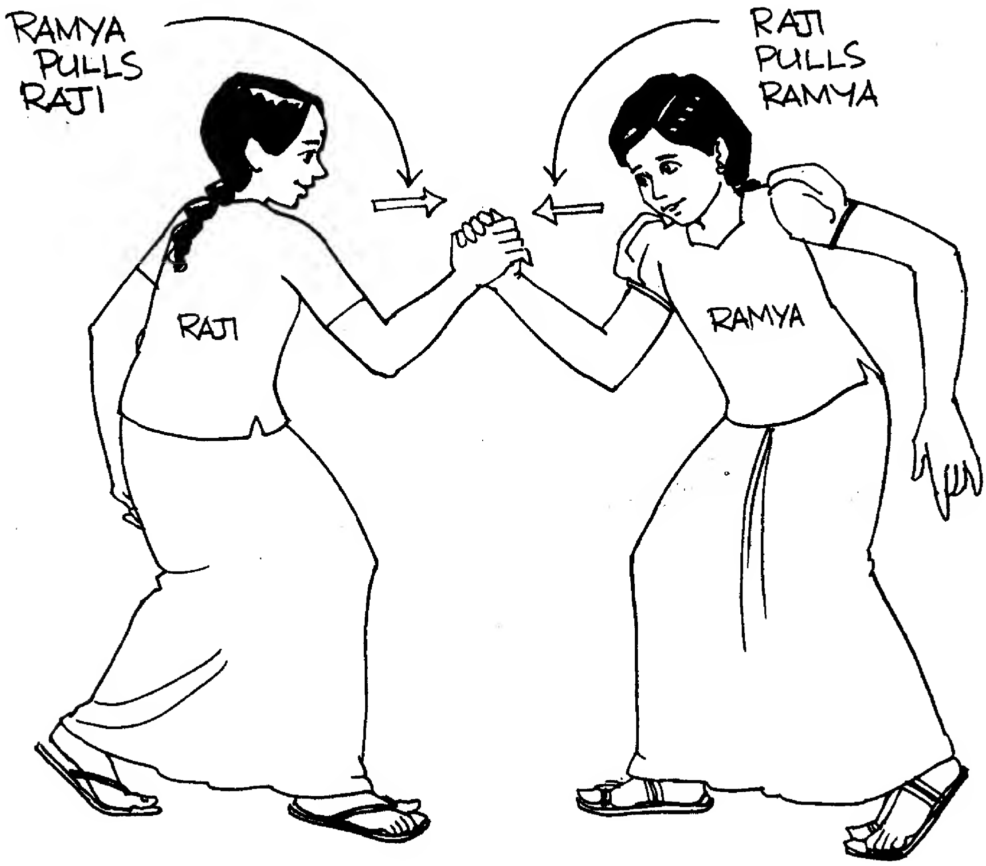

WHO IS THE STRONGEST IN THE CLASS?
VIKRAM!
WHO IS THE WEAKEST?
VIJAY!
IF WE HAVE A TUG OFF WAR BETWEEN THEM WHO WILL WIN?
VIKRAM

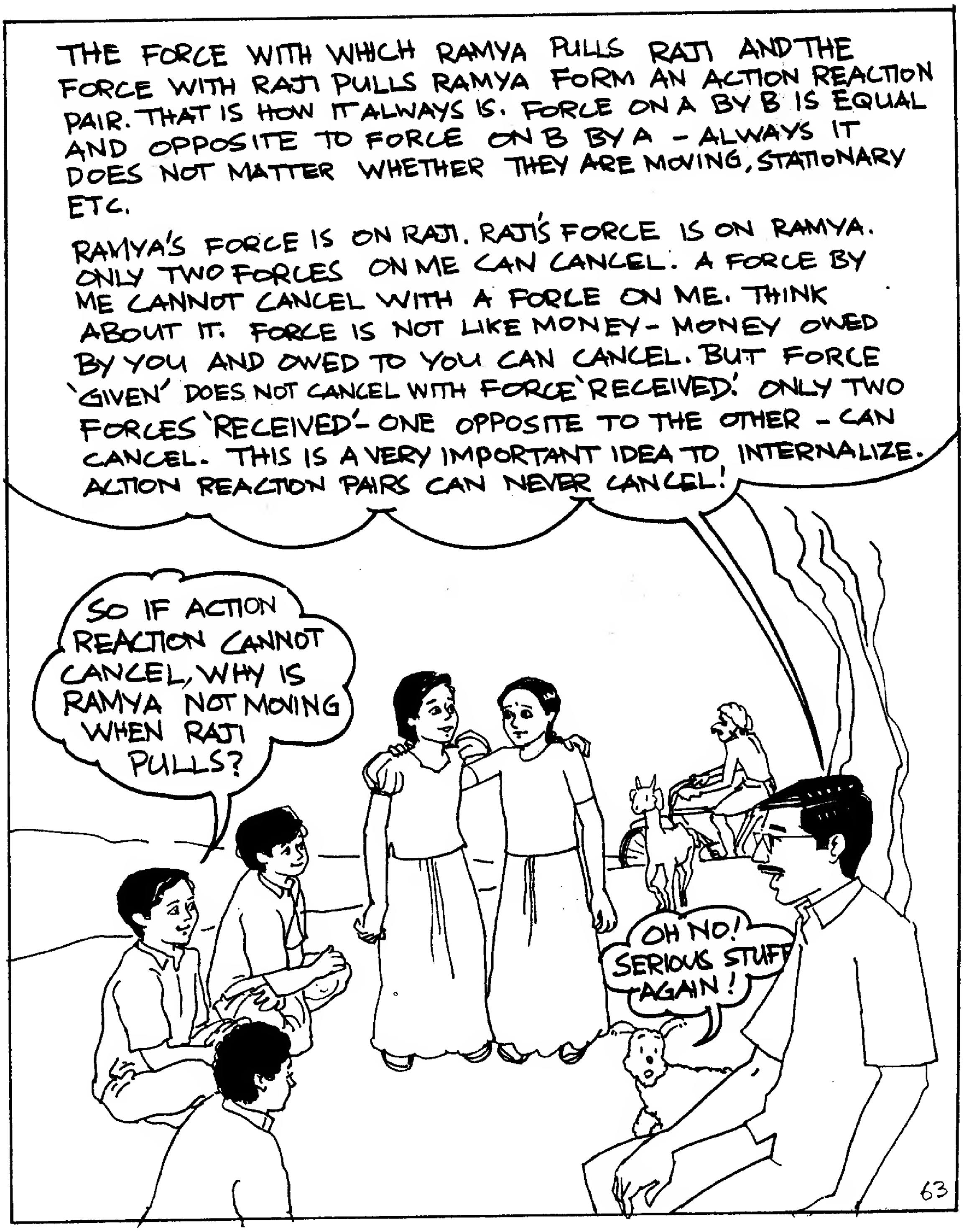

LET'S SEE. BOTH OF YOU START PULLING. BUT VIKRAM, YOU SHOULD KEEP YOUR LEGS TOGETHER. NOT APART LIKE YOU ARE KEEPING NOW. YES - LIKE THAT AND KEEP YOUR BODY STRAIGHT - DON'T BEND IT BACKWARDS. OK NOW PULL.
VIJAY WHO IS WEAKER PULLS VIKRAM EASILY
SO WHO WON?
VIJAY. BUT IT WAS UNFAIR VIKRAM'S FEET WERE TOGETHER.
BUT WHY DOES THAT MATTER?
HE CAN'T BALANCE!


WHO ASKED VIKRAM TO BALANCE? I ONLY ASKED HIM TO PULL! AND HE LOST. SO SHALL WE SAY VIKRAM IS WEAKER THAN VIJAY WHO IS THE WEAKEST IN CLASS.
HA! HA!
OK, SERIOUSLY. WHY WAS VIKRAM NOT ABLE TO PULL HARD? WHAT HAS FOOT GOT TO DO WITH HAND PULLING? LET'S SEE HOW I PULL VIKRAM. LOOK AT MY LEGS. HOW ARE THEY PLACED?
ONE LEG IS FORWARD. AND ANOTHER BEHIND.
WITH WHICH LEG AM I PUSHING MORE?
FRONT LEG!
OK, NOW ALL OF YOU STAND UP AND PULL IN PAIRS.
60


THE STUDENTS COME OUT, PULL EACH OTHER AND HAVE A LOT OF FUN...
CAN YOU FEEL YOUR FRONT LEG PUSHING?
YES!
NOW THE REST OF YOU GO AND SITDOWN. YOU BOTH CONTINUE TO PULL...
AS YOU ARE PULLING LIFT YOUR BACK LEG AWAY FROM THE GROUND
NOW PUT YOUR BACK LEG DOWN AND LIFT YOUR FRONT LEG...


WHY IS RAMYA NOT ABLE TO LIFT HER FRONT LEG?
SHE WILL LOSE BALANCE
NOT JUST THAT. IT IS THE FRONT LEG WITH WHICH SHE IS PUSHING THE GROUND - SO THAT RAJI DOES NOT PULL HER AWAY. IF RAMYA LIFTS HER FRONT LEG SHE STARTS MOVING TOWARDS RAJI. LET'S UNDERSTAND THIS IN MORE DETAIL.


RAMYA PULLS RAJI
RAJI PULLS RAMYA
RAJI
RAMYA



THE FORCE WITH WHICH RAMYA PULLS RAJI AND THE FORCE WITH RAJI PULLS RAMYA FORM AN ACTION REACTION PAIR. THAT IS HOW IT ALWAYS IS. FORCE ON A BY B IS EQUAL AND OPPOSITE TO FORCE ON B BY A - ALWAYS IT DOES NOT MATTER WHETHER THEY ARE MOVING, STATIONARY ETC.
RAMYA'S FORCE IS ON RAJI. RAJI'S FORCE IS ON RAMYA. ONLY TWO FORCES ON ME CAN CANCEL. A FORCE BY ME CANNOT CANCEL WITH A FORCE ON ME. THINK ABOUT IT. FORCE IS NOT LIKE MONEY - MONEY OWED BY YOU AND OWED TO YOU CAN CANCEL. BUT FORCE 'GIVEN' DOES NOT CANCEL WITH FORCE 'RECEIVED.' ONLY TWO FORCES 'RECEIVED' - ONE OPPOSITE TO THE OTHER - CAN CANCEL. THIS IS A VERY IMPORTANT IDEA TO INTERNALIZE. ACTION REACTION PAIRS CAN NEVER CANCEL!
SO IF ACTION REACTION CANNOT CANCEL, WHY IS RAMYA NOT MOVING WHEN RAJI PULLS?
OH NO! SERIOUS STUFF AGAIN!
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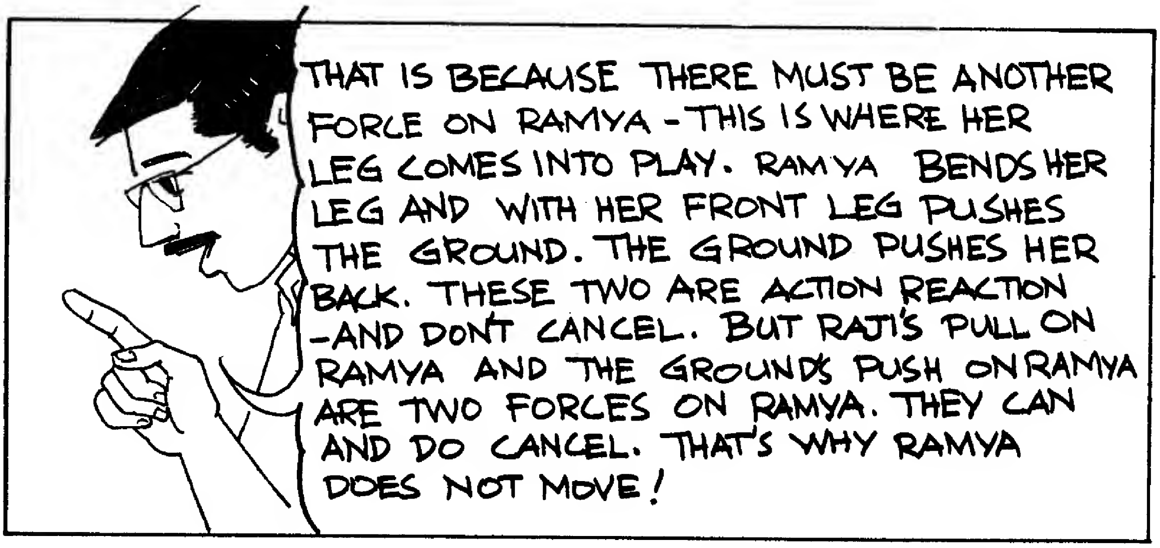
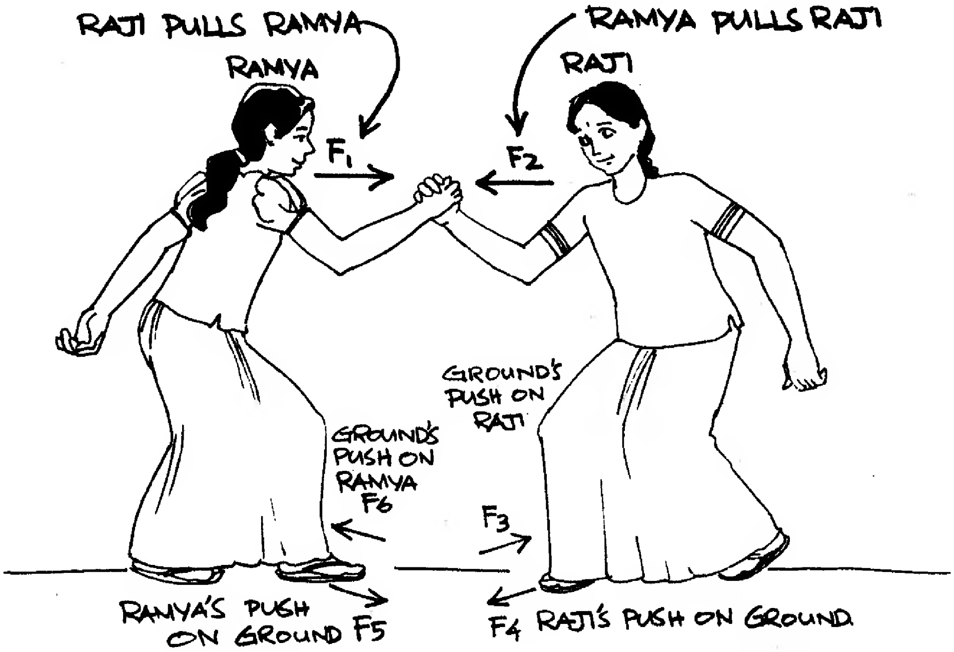
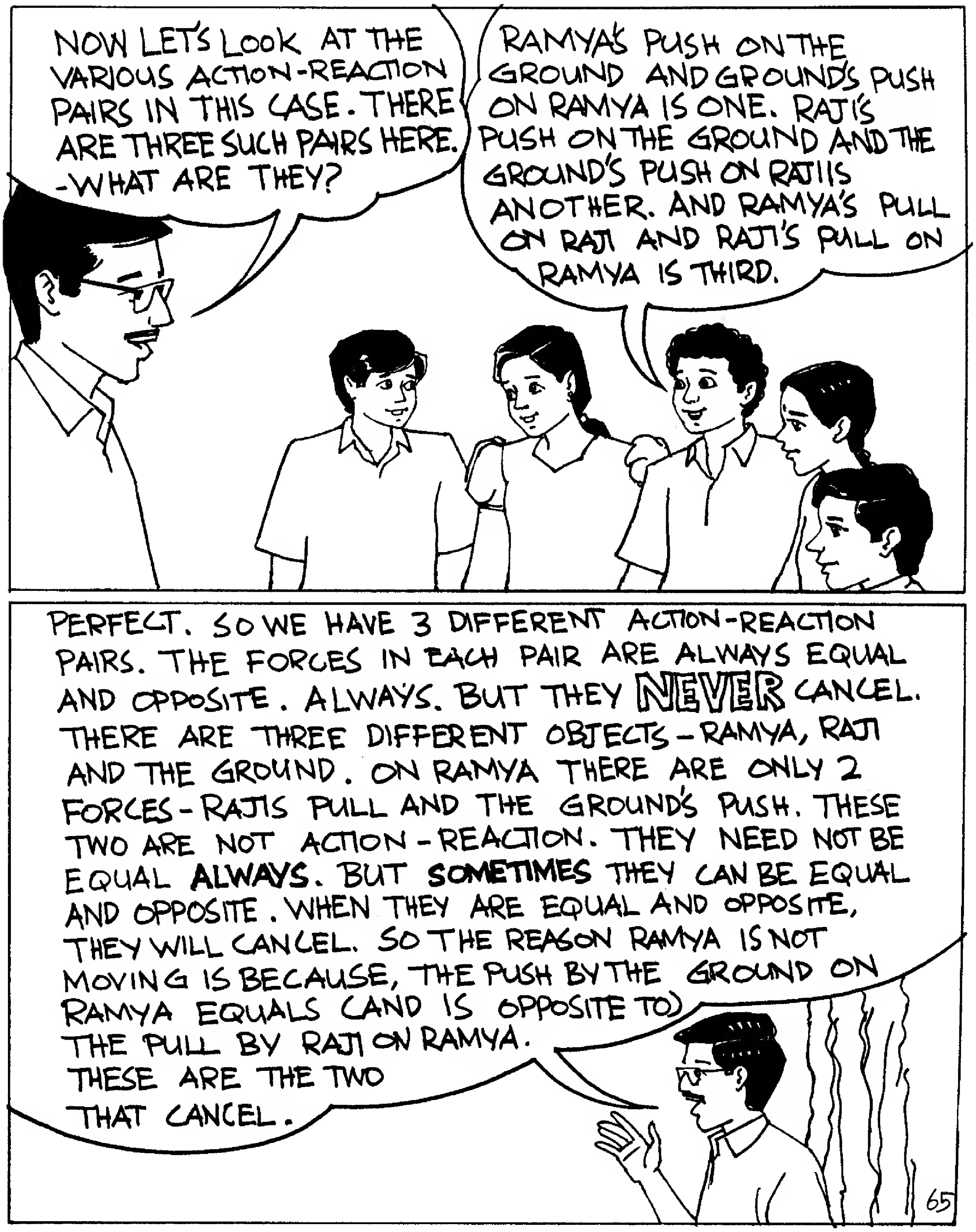




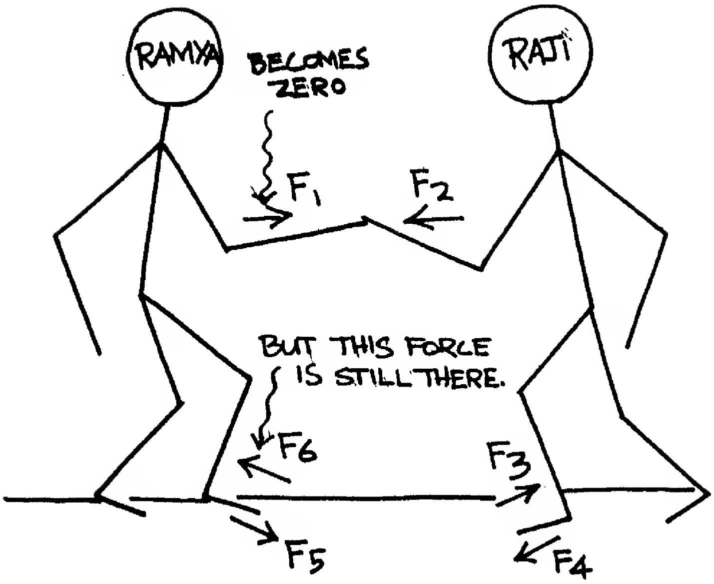
IT IS NOT RAJI'S PULL ON RAMYA AND RAMYA'S PULL ON RAJI THAT CANCEL. IT IS RAJI'S *PULL ON RAMYA* AND *THE GROUNDS PUSH ON RAMYA* CANCELLING EACH OTHER. THAT'S WHY RAMYA DOES NOT MOVE.



NOW LET'S LOOK AT THE VARIOUS ACTION-REACTION PAIRS IN THIS CASE. THERE ARE THREE SUCH PAIRS HERE. -WHAT ARE THEY?
RAMYA'S PUSH ON THE GROUND AND GROUND'S PUSH ON RAMYA IS ONE. RAJI'S PUSH ON THE GROUND AND THE GROUND'S PUSH ON RAJI IS ANOTHER. AND RAMYA'S PULL ON RAJI AND RAJI'S PULL ON RAMYA IS THIRD.
PERFECT. SO WE HAVE 3 DIFFERENT ACTION-REACTION PAIRS. THE FORCES IN EACH PAIR ARE ALWAYS EQUAL AND OPPOSITE. ALWAYS. BUT THEY NEVER CANCEL. THERE ARE THREE DIFFERENT OBJECTS - RAMYA, RAJI AND THE GROUND. ON RAMYA THERE ARE ONLY 2 FORCES - RAJI'S PULL AND THE GROUND'S PUSH. THESE TWO ARE NOT ACTION-REACTION. THEY NEED NOT BE EQUAL ALWAYS. BUT SOMETIMES THEY CAN BE EQUAL AND OPPOSITE. WHEN THEY ARE EQUAL AND OPPOSITE, THEY WILL CANCEL. SO THE REASON RAMYA IS NOT MOVING IS BECAUSE, THE PUSH BY THE GROUND ON RAMYA EQUALS (AND IS OPPOSITE TO) THE PULL BY RAJI ON RAMYA. THESE ARE THE TWO THAT CANCEL.

THE FORCE PAIRS THAT ARE EQUAL ARE F1=F2, F3=F4, F5=F6 DOES THIS MEAN, ALL HAVE TO BE EQUAL TO EACH OTHER?
NO F1 AND F2 CAN BE 1, F3 AND F4 CAN BE 2, AND F5 AND F6 CAN BE 3
RIGHT. BUT IS IT POSSIBLE TO HAVE FOUR OF THEM EQUAL AND OTHER TWO DIFFERENT.
YES. F1 = F2 = F3 = F4 = 2 AND F5 = F6 = 1 IS POSSIBLE.
F1 F2
F6 F3
F5 F4
THAT'S CORRECT. THE ONLY CONDITION NEWTON'S THIRD LAW PLACES IS THAT EACH ACTION-REACTION PAIR HAS BOTH FORCES EQUAL - THEY NEED NOT BE EQUAL TO FORCES IN OTHER ACTION REACTION PAIRS. F1 IS ALWAYS EQUAL TO F2. BUT SOMETIMES F1 IS ALSO EQUAL TO F6 (WHICH IS ALWAYS EQUAL TO F5). WHEN THAT HAPPENS, RAMYA DOES NOT MOVE. IF RAMYA MOVES TOWARDS RAJI - THEN IT DOES NOT MEAN F1 IS MORE THAN F2. IT MEANS F1 IS MORE THAN F6. RAJI'S FORCE ON RAMYA IS MORE THAN GROUND'S FORCE ON RAMYA. RAMYA IS UNABLE TO HOLD BACK BY PUSHING THE GROUND HARD ENOUGH - THAT'S WHY THE GROUND PUSHES HER LESS AND RAJI IS ABLE TO PULL RAMYA AWAY. ON THE OTHER HAND, RAJI CAN EXERT ENOUGH FORCE ON THE GROUND AND THE GROUND EXERTS FORCE ON RAJI. AND SO RAJI DOES NOT MOVE.
IN THIS CASE F1 = F2 = F3 = F4 AND THEY ARE ALL GREATER THAN F5 AND F6


NOW LET'S DO ANOTHER EXPERIMENT. BOTH OF YOU KEEP PULLING EACH OTHER.
RAJI
RAMYA
WHEN I SAY 'YES' YOU SUDDENLY LET RAMYA'S HANDS GO!
BOTH OF YOU PULL HARD...
YES...!
WHY DID RAMYA FALL BACK?
BECAUSE RAJI STOPPED PULLING SUDDENLY!
YELP! YELP!
LET'S TRY TO UNDERSTAND THIS WITH OUR OTHER IDEAS LEARNT EARLIER.

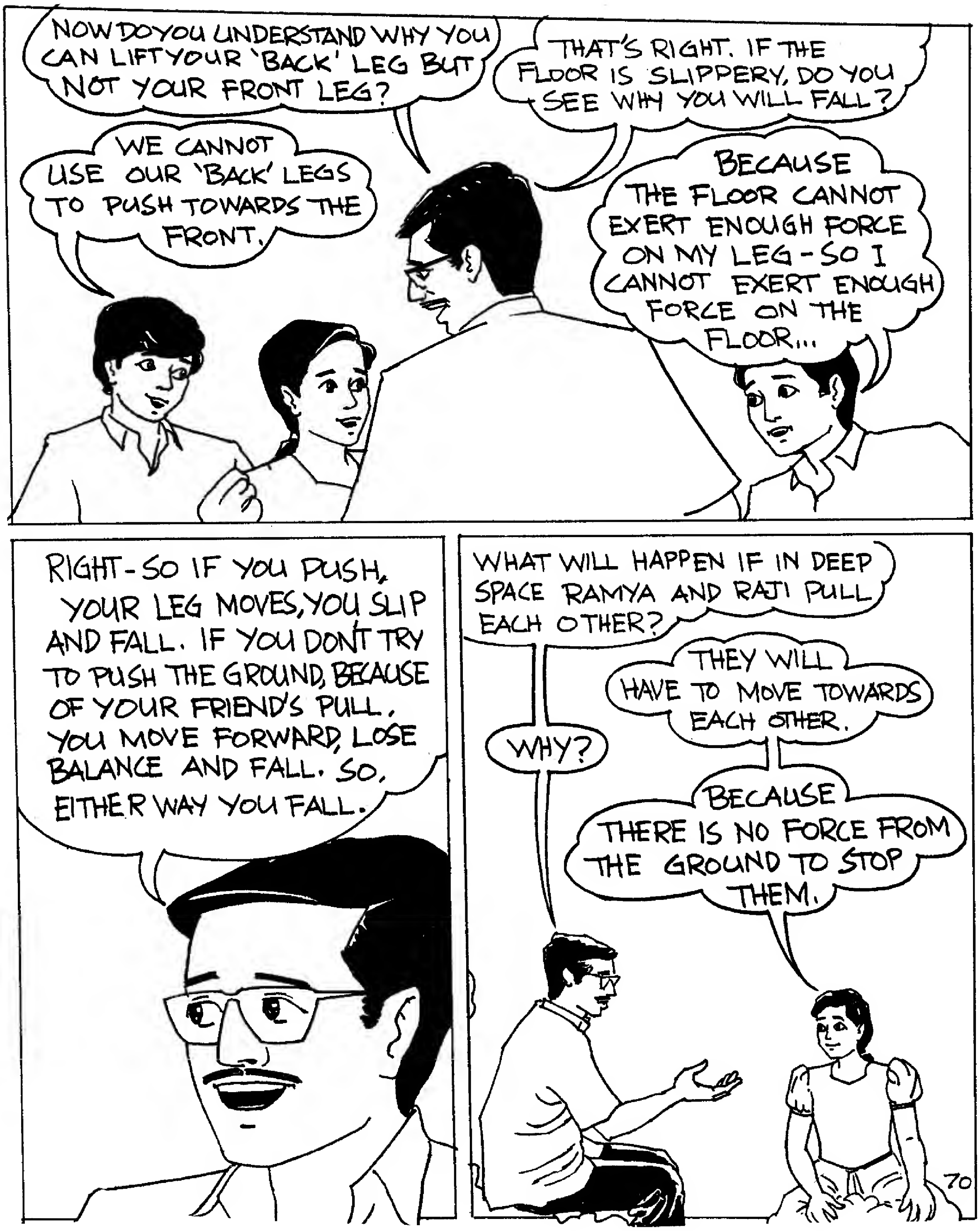


Raji stopped pulling Ramya. So F1 (the force on Ramya) becomes zero. Ramya did not know Raji is going to stop pulling. Only after Raji's force stops, Ramya's hand knows about it and only after this her hand sends a signal informing her brain that there is not force on her hand. Then her brain sends another signal to her legs, telling her legs to stop pushing the ground. All this takes time. Till then her legs continue pushing and the ground continues pushing her. During this time there is only one force on Ramya - ground pushing her (F6). F1 has suddenly become zero. That's why Ramya falls backward.

S: But if Raji stops pulling Ramya, doesn't Ramya's pull on Raji (F2) also become zero at the same time? Why then doesn't Raji also fall down?

T: Yes, Ramya's pull on Raji is always equal to Raji's pull on Ramya. This means F2 is also zero. But Raji knows she is going to stop pulling. So her brain tells her legs in advance to stop reducing her push on the ground and F4 becomes zero. The ground stops pushing Raji (F3 = 0). Earlier, Raji did not move because both non-zero forces (F2 and F3) cancelled each other. Now, Raji does not move (or fall) because both forces are zero (and therefore the net force is still zero)!


WHEN RAJI RELEASES HER HAND SLOWLY AND NOT SUDDENLY - THEN RAMYA'S LEG GETS THE MESSAGE BEFORE RAJI'S PULL ON HER BECOMES ZERO AND HER LEG REDUCES IT'S PUSH AND PREVENTS HER FROM FALLING BACK. SO WHEN RAJI DOES THIS AFTER TELLING RAMYA OR DOES IT SLOWLY, RAMYA DOESN'T FALL BACK.
ALL OF YOU COME OUT AND PULL EACH OTHER - THIS TIME CAREFULLY START PULLING AND SEE HOW YOUR LEGS RESPOND - CAN YOU SEE YOU ARE EXERTING MORE FORCE AS YOUR FRIEND IS PULLING MORE?
YES!
YES!
YOUR LEGS ARE RESPONDING TO YOUR BRAIN'S INSTRUCTION. THE BRAIN GETS A SIGNAL FROM YOUR HAND GIVING INFOR-MATION ABOUT THE AMOUNT OF FORCE ON IT, THE BRAIN THEN TELLS THE LEG TO EXERT ENOUGH FORCE TO STOP YOU FROM FALLING. THIS IS A CONTINOUS FEEDBACK SYSTEM. WITHOUT THIS YOU CANNOT COORDINATE BETWEEN DIFFERENT PARTS OF YOUR BODY.


NOW DO YOU UNDERSTAND WHY YOU CAN LIFT YOUR 'BACK' LEG BUT NOT YOUR FRONT LEG?
WE CANNOT USE OUR 'BACK' LEGS TO PUSH TOWARDS THE FRONT.
THAT'S RIGHT. IF THE FLOOR IS SLIPPERY, DO YOU SEE WHY YOU WILL FALL?
BECAUSE THE FLOOR CANNOT EXERT ENOUGH FORCE ON MY LEG - SO I CANNOT EXERT ENOUGH FORCE ON THE FLOOR...
RIGHT - SO IF YOU PUSH, YOUR LEG MOVES, YOU SLIP AND FALL. IF YOU DON'T TRY TO PUSH THE GROUND, BECAUSE OF YOUR FRIEND'S PULL, YOU MOVE FORWARD, LOSE BALANCE AND FALL. SO, EITHER WAY YOU FALL.
WHAT WILL HAPPEN IF IN DEEP SPACE RAMYA AND RAJI PULL EACH OTHER?
THEY WILL HAVE TO MOVE TOWARDS EACH OTHER.
WHY?
BECAUSE THERE IS NO FORCE FROM THE GROUND TO STOP THEM.

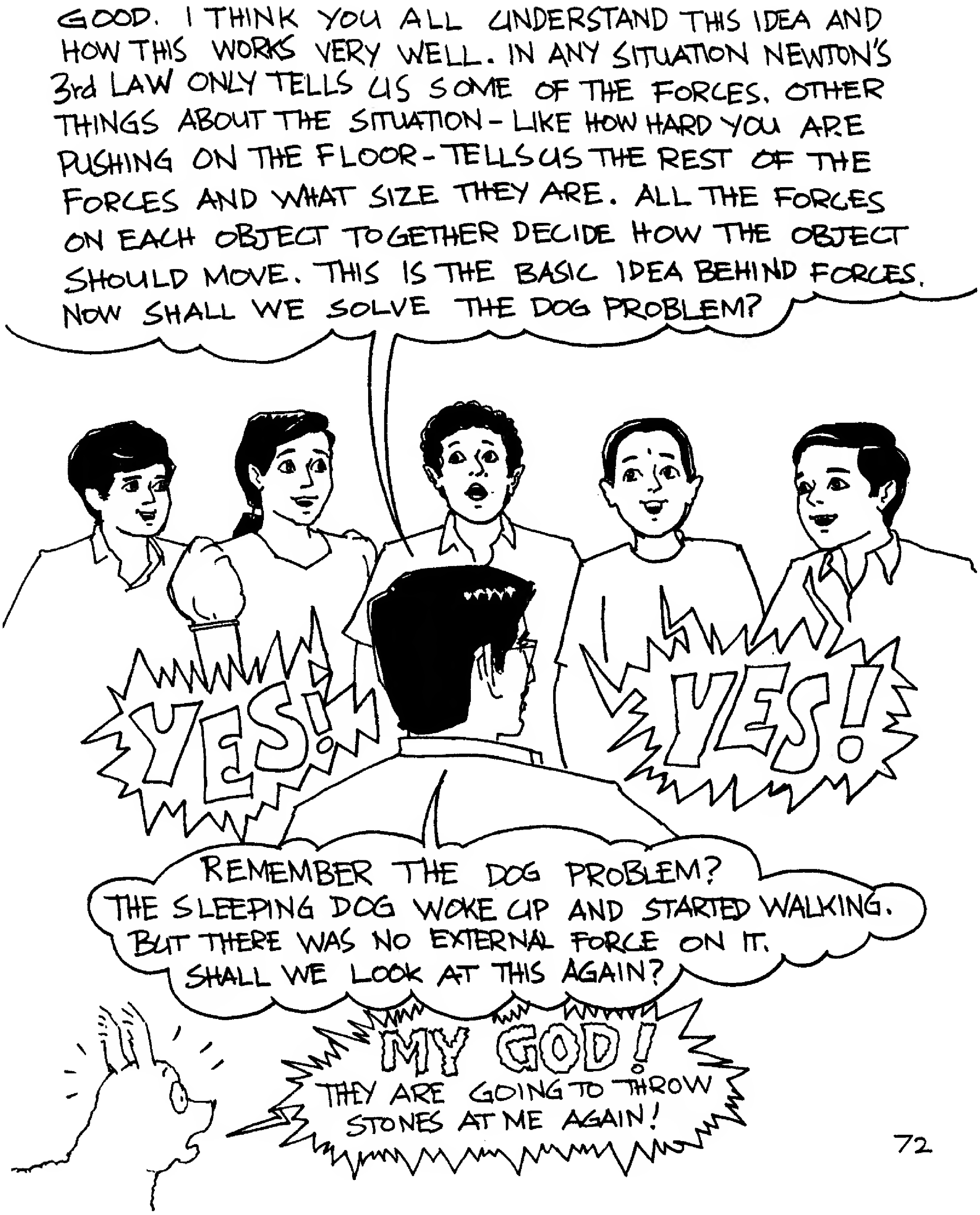
Chapter 1: Section D

Back to Dog Walking !

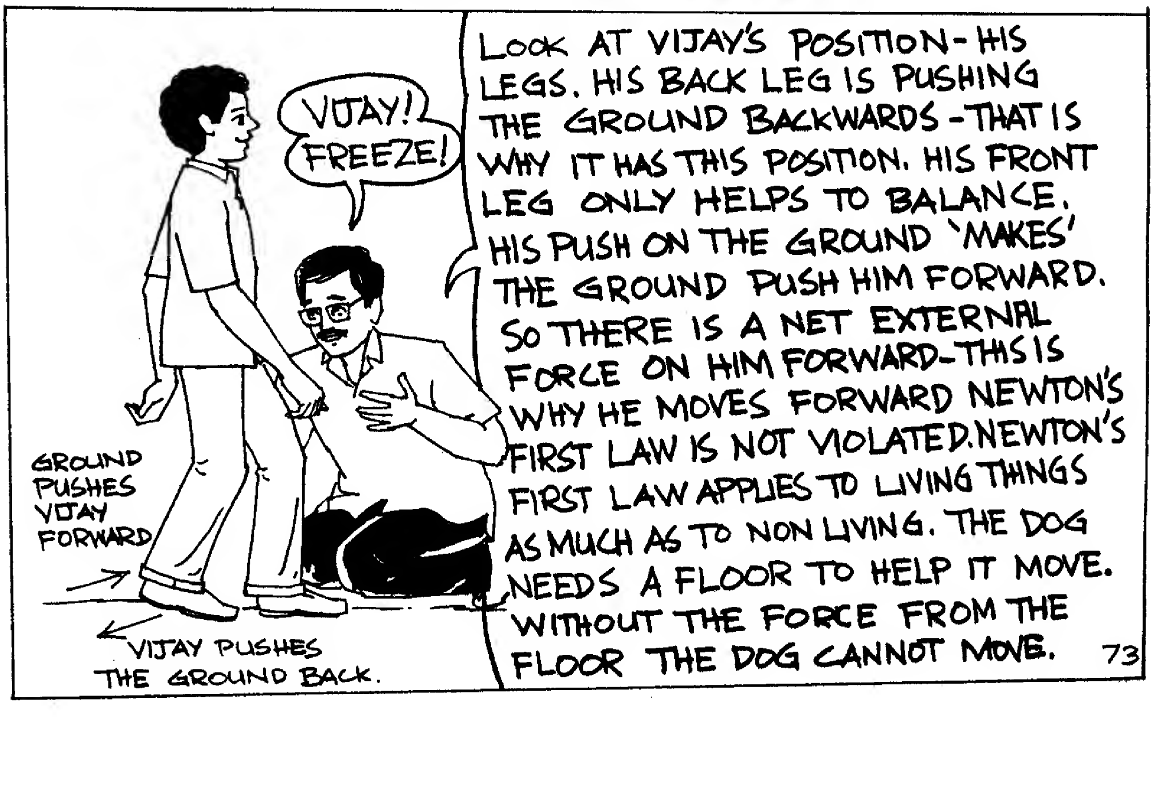
This is the last section in this chapter. Armed with a clear understanding of Newton's third law, we finally return to tackle the case of the dog walking.

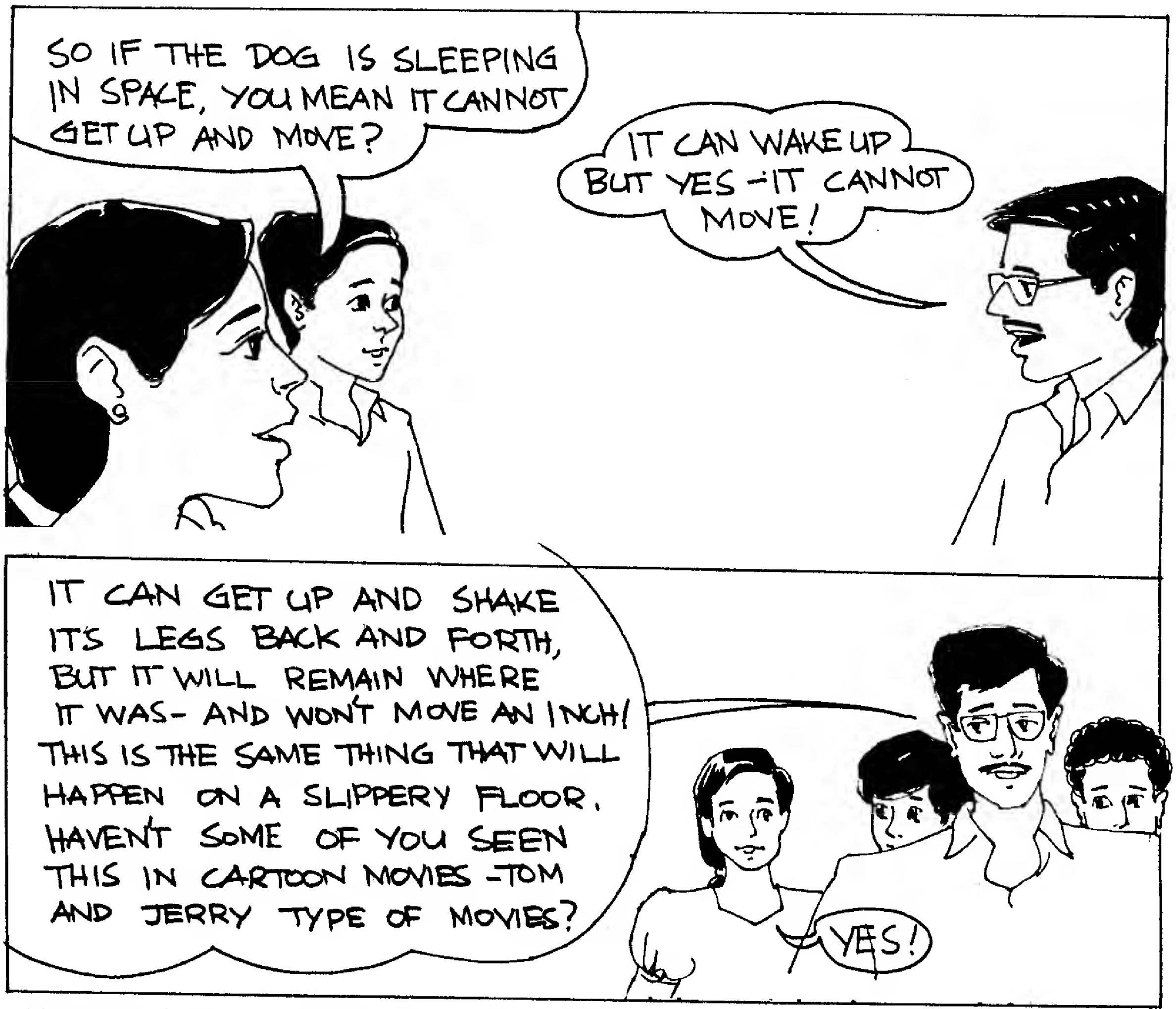
In the previous section, we saw how legs play a critical role in a tug of war. Legs also play a critical role in walking. But as is popularly assumed, it is not the force from our legs force that pushes us forward. That is not an external force. To be able to walk, we need the earth to push us forward. It is only by pushing the earth backward, a dog is able to walk.

Newton's first law works in the dog's case as well. This means a dog or a human cannot start walking or moving on their own in deep space. You need to push another object away every time you want to move. On the earth, the earth itself is a convenient object that we can push and walk. With this understanding the reader can also understand, why we can stand on an oily floor, but when we try to walk, we invariably slip and fall.


GOOD. I THINK YOU ALL UNDERSTAND THIS IDEA AND HOW THIS WORKS VERY WELL. IN ANY SITUATION NEWTON'S 3rd LAW ONLY TELLS US SOME OF THE FORCES. OTHER THINGS ABOUT THE SITUATION - LIKE HOW HARD YOU ARE PUSHING ON THE FLOOR - TELLS US THE REST OF THE FORCES AND WHAT SIZE THEY ARE. ALL THE FORCES ON EACH OBJECT TOGETHER DECIDE HOW THE OBJECT SHOULD MOVE. THIS IS THE BASIC IDEA BEHIND FORCES. NOW SHALL WE SOLVE THE DOG PROBLEM?
YES!
YES!
REMEMBER THE DOG PROBLEM? THE SLEEPING DOG WOKE UP AND STARTED WALKING. BUT THERE WAS NO EXTERNAL FORCE ON IT. SHALL WE LOOK AT THIS AGAIN?
MY GOD! THEY ARE GOING TO THROW STONES AT ME AGAIN!


OH! THE DOG HAS RUN AWAY! DOESN'T MATTER. LET'S UNDERSTAND VIJAY'S WALKING INSTEAD
VIJAY, PLEASE START WALKING!
HA!
HA! HA!


VIJAY! FREEZE!
LOOK AT VIJAY'S POSITION- HIS LEGS. HIS BACK LEG IS PUSHING THE GROUND BACKWARDS - THAT IS WHY IT HAS THIS POSITION. HIS FRONT LEG ONLY HELPS TO BALANCE. HIS PUSH ON THE GROUND 'MAKES' THE GROUND PUSH HIM FORWARD. SO THERE IS A NET EXTERNAL FORCE ON HIM FORWARD- THIS IS WHY HE MOVES FORWARD. NEWTON'S FIRST LAW IS NOT VIOLATED. NEWTON'S FIRST LAW APPLIES TO LIVING THINGS AS MUCH AS TO NON LIVING. THE DOG NEEDS A FLOOR TO HELP IT MOVE. WITHOUT THE FORCE FROM THE FLOOR THE DOG CANNOT MOVE.
GROUND PUSHES VIJAY FORWARD
VIJAY PUSHES THE GROUND BACK.



Newton's three laws together form one package - you need the third law to completely understand the first and similarly you need the first law to understand the second and so on. It is not as if the first law can be understood first and then the second and then the third. You need all the laws to understand each of the laws. This co-dependence is not just true of Newton's laws - it is true of all of science. Every part of science is linked to other parts in many ways - we cannot understand one part without understanding other parts. We have to see the three laws as steps in understanding one unified nature. The steps are ours - nature is one.

Laws like these are very deep. Only by applying them to practical situations and thinking hard about what they mean can we fully understand them. It is not a question of memorizing them and repeating them. You have to think about how they apply in many situations to actually understand what such a simple statement like Action equal Reaction means. This process can be fun. With this we come to the end of this chapter. Below is a summary of the main ideas that we have discussed in this chapter.

Key Ideas in this Lecture

1. Newton's laws of motion are applicable to all objects - living, non-living, thinking, non-thinking - all objects.

2. There are two parts to Newton's first law - when the object is at rest and in motion. The rest part of the law looks obvious - but is really not so. Even in simple cases like the dog example it takes a lot of thinking and understanding of the third law to understand why it moves. We have not looked at the bodies in motion yet. We will do this in the next chapter.

3. What is Force ? Intuitively - force is a push. But we should be careful not to confuse pushing with moving - a lot of confusion arises because of this.

4. Action and Reaction are really only forces - Action in Physics is not 'action' in ordinary language. Action is the force exerted by one body on another. Reaction is again force exerted by the second body on the first. Newton's third law says these two are equal. We should think of the situation in terms of 3 words and ideas - Action, Reaction and Response and not the usual two - Action and Reaction.

5. Action and Reaction are **not** cause and effect. Without the reaction, there can be no action - Reaction is the resistance which allows us to exert the force - both forces happen simultaneously and neither is the cause of the other. Action (or Reaction) causes the Response. We can arbitrarily choose which of the two forces to call Action - the other will automatically become the Reaction. You **cannot** exert a force without yourself feeling an equal and opposite force.

6. There is a tendency to confuse Force and Response. They are two different ideas. Different response does not mean different force. The same force can yield different responses on different objects. But the reaction to the same force is always the same in all cases. Response depends on many factors - force is **only one** of them.

7. The idea of force means that if the same force acts on the same object (and nothing else changes) - then the object will have the same response. But how different objects respond to the same force can differ.

8. Then we looked at three laws that are at the foundation of the Newtonian framework. These are laws that are assumed when Newton's laws are discussed. They sound simple - but are not obvious and it is important to state and understand them clearly.

   a. **Newton's Zeroth Law:** Every particle which acts or disturbs another particle does it through a vector quantity called Force. The entire action of the first particle on the second is completely determined by the force it exerts on the second. We can replace the first particle by its force. This means there is only one force exerted by an object on another. (sometimes we think of this one force in terms of its components - then we can think of 2 forces).

A More Advanced Idea: In most of what we discussed we talked to removing the entire object itself. In reality, we ascribe forces to properties of an object - the force arises because of its mass, or because of its charge, its magnetic moment etc - When we talk of replacing the object by its force - we usually mean only replacing this property by its force. So we talk of gravitational forces on an object and mentally remove the mass causing the gravitational force. But this is only a small step from what was discussed.

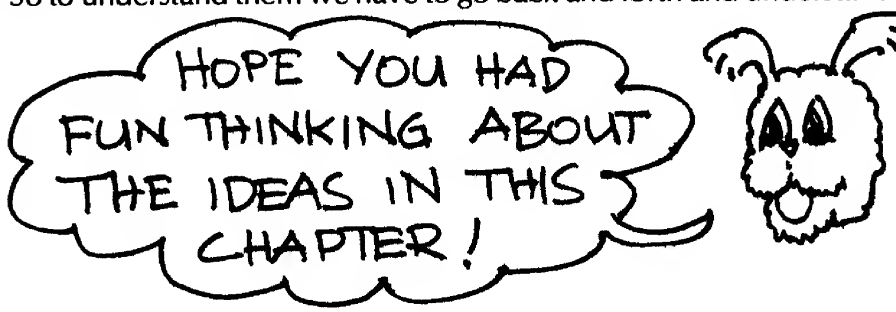
b. **Newton's Half-th Law:** Each particle acting on another particle does so independently of other particles. This means the force exerted by a particle on another does not depend on what other particles nearby are doing. The net force on a particle is the resultant of all the forces on the particle by all the other particles.

c. **Newton's three-fourth Law:** If there are several forces on a particle, it behaves as if there is only one net force - this net force can be found by a special addition called vector addition of forces. We can add forces like arrows.

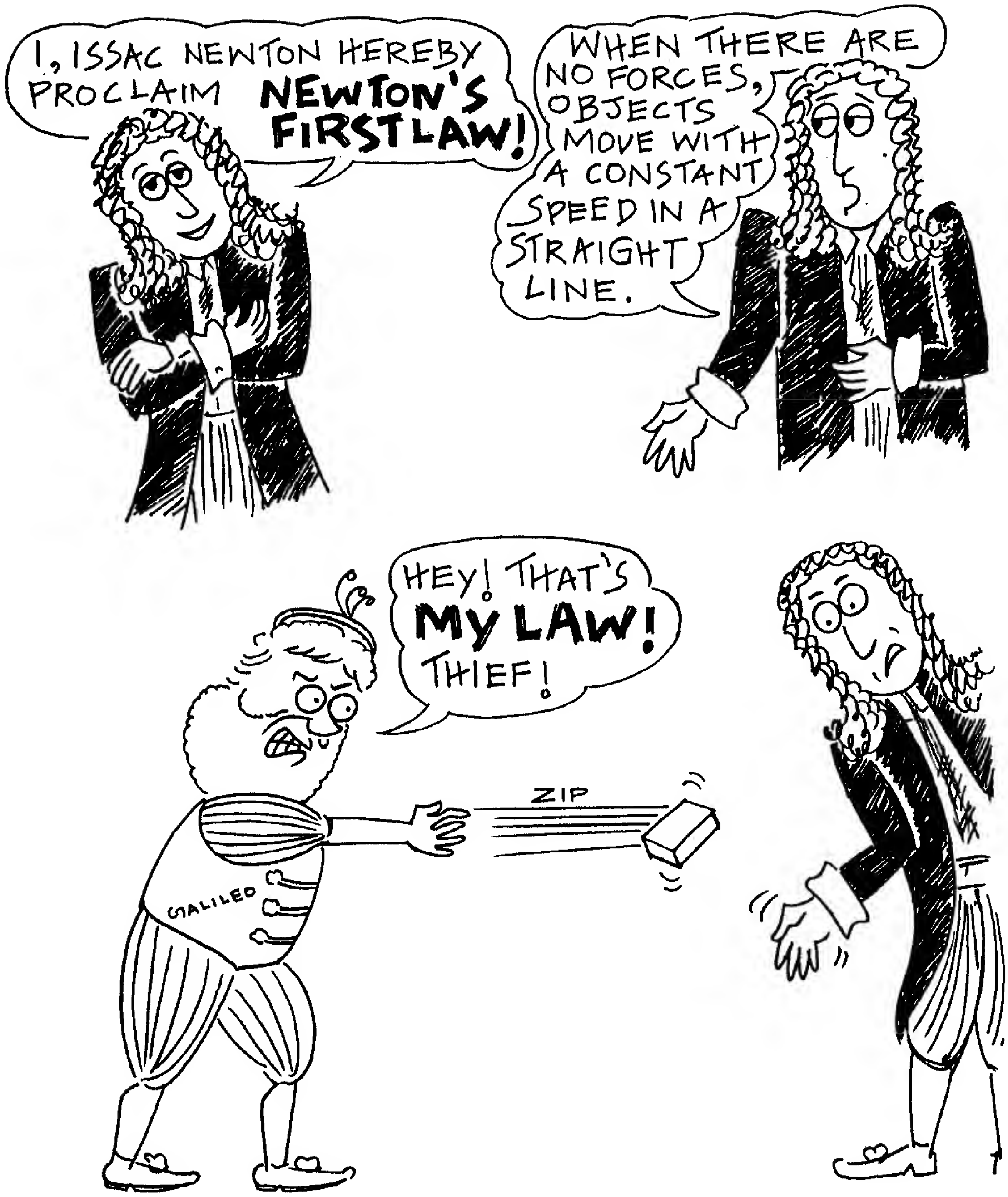
   This idea has to be qualified - this is absolutely true only for ideal particles. On most objects, forces can have other effects. Force not only makes things move, they also change shape, cause pain etc. This net force idea (therefore the idea of two equal and opposite forces canceling) is true for particles and for other objects only as far as 'net movement' of the object is concerned.

9. **Cancellation of two Forces**: Only forces **on** the same object can cancel. Forces on an object cannot cancel with forces by the object on other objects. Therefore Action and Reaction can never cancel. Action and Reaction is always **on** and **by** an object. **Action and Reaction are always equal** - even when things move. If something is not moving then it possibly has to be two different forces **on** the object that cancel. These two forces cannot be an Action-Reaction pair.

10. **Newton's laws are one package** - they are part of one complete plan. They together form a framework through which we can think about and understand the world. Without one law you cannot understand the other laws - only together make sense. So to understand them we have to go back and forth and understand them together.



Chapter 2
Newton's First Law



Chapter 2 - Section A

Newton's First Law - The Basics

In the previous chapter we looked at what really is meant by objects at rest continuing to remain at rest. We also saw the deep connection between the third law and the first law. In this chapter we will look at the second part of the first law - objects moving uniformly forever on their own.

The first section is a long one. Here we explore the meaning of the moving forever part of Newton's first law. The large part of this section is an imaginary dialogue between Aristotle and Galileo enacted by students and the teacher. The main argument of this section is very easy to state: "if there is no net external force, a body keeps moving uniformly forever". Then why have such a long section? Because our purpose here is really to see how this law came about and what is the basis for it. Our aim is not merely understanding the statement but rather understanding the rationale behind the statement. Through this process, we hope the reader gets acquainted with the process of doing science - the combination of experiments, arguments and logic that makes science.

This section starts with questioning the validity of "objects naturally moving forever". We then move on to ask what is friction and how we know it exists. Then comes an argument between Aristotle and Galileo - the former proposing that things naturally stop and there is nothing called friction and the latter arguing that things tend to move forever and if a moving object stops, it is because a force stops it. This argument goes on with debates and experiments. Finally Galileo wins the argument and establishes that things naturally tend to move uniformly forever.


IN THE PREVIOUS CHAPTER, YOU TALKED ABOUT WHY OBJECTS AT REST REMAIN AT REST WITHOUT AN EXTERNAL FORCE AND WHY A DOG IN SPACE CANNOT GET UP AND START WALKING ON ITS OWN. BUT NEWTON'S FIRST LAW ALSO SAYS THAT MOVING OBJECTS KEEP ON MOVING. YOU DID NOT SAY WHY THAT IS SO.
OK. LET'S DISCUSS THAT NOW. WHAT DOES THIS PART OF NEWTON'S LAW REALLY SAY?
UNIFORMLY FOREVER
OBJECTS THAT ARE MOVING, KEEP MOVING UNIFORMLY IN A STRAIGHT LINE FOREVER WHEN THERE IS NO EXTERNAL FORCE.
BUT DO YOU BELIEVE IN THIS LAW?
YES. THEY WILL KEEP MOVING IF THERE IS NO FRICTION.
NO- MOVING THINGS WILL FINALLY COME TO STOP HOW CAN THEY KEEP ON MOVING FOREVER?
79

T: Ok, how do we know moving things will keep on moving?

S: They tested it and found out.

T: Who tested it?

S: Scientists.

T: How do you know? What did they really test? Will you believe anything a 'scientist' tells you? What about your own ideas of the world?

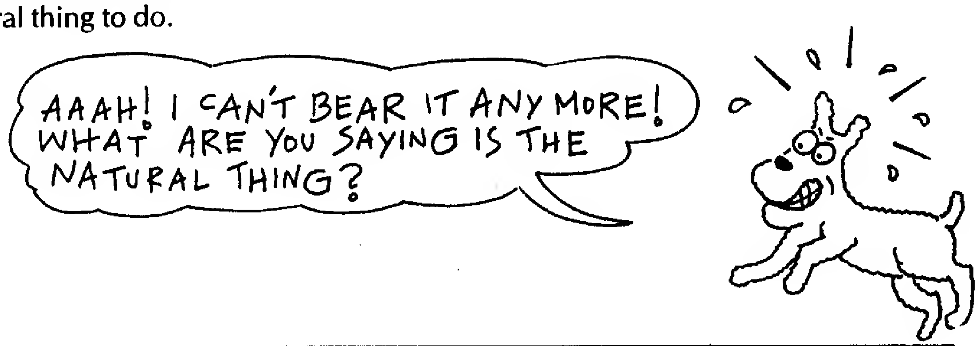
T: Have you seen objects at rest remain at rest when they are not disturbed?

S: Yes.

T: So you can believe the first part of Newton's law. When things start moving or falling, we usually look for a source of disturbance.


WHEN HAVE WE SEEN OBJECTS MOVING FOREVER WITHOUT STOPPING? IF I PUSH THIS DUSTER AND THEN REMOVE MY HAND WILL IT KEEP MOVING ON THE FLOOR FOREVER?
IT WILL STOP
NO. IT STOPS BECAUSE OF FRICTION. IN FRICTIONLESS SURFACES IT WILL KEEP MOVING FOREVER
EVEN AMONGST YOU THERE ARE DIFFERENCES. SOME OF YOU BELIEVE IT REALLY STOPS. OTHERS BELIEVE THERE IS SOMETHING CALLED FRICTION THAT STOPS IT. I AM GOING TO SIDE WITH THOSE WHO BELIEVE NEWTON'S LAW IS WRONG! MOVING DUSTERS STOP. I SEE IT ALL THE TIME. I CANNOT SEE FRICTION. HOW DO YOU KNOW THERE IS FRICTION? HOW DO YOU KNOW IT IS NOT NATURAL FOR OBJECTS TO STOP MOVING?
??
??
???

T: What we are discussing is hard - very hard. You have to follow the argument slowly and carefully to understand it. When you say a moving object left to itself will stop - you are saying that stopping is the natural thing to do. When you say on the other hand that a moving object left to itself will keep on moving forever in a straight line, you are saying that moving forever is the natural thing for it to do. The question is which of these is really the object's natural thing to do.



The real question...

What will an object do naturally ? Is it natural for a moving object to stop or to continue moving?

Is there Friction ?

What does this really mean? You cannot see something and someone says it exists - you lie back and say "aha, friction exists." But knowing a name is not the same as knowing an idea. When you only know friction exists - *don't fool yourself*, you only know a name - nothing more.

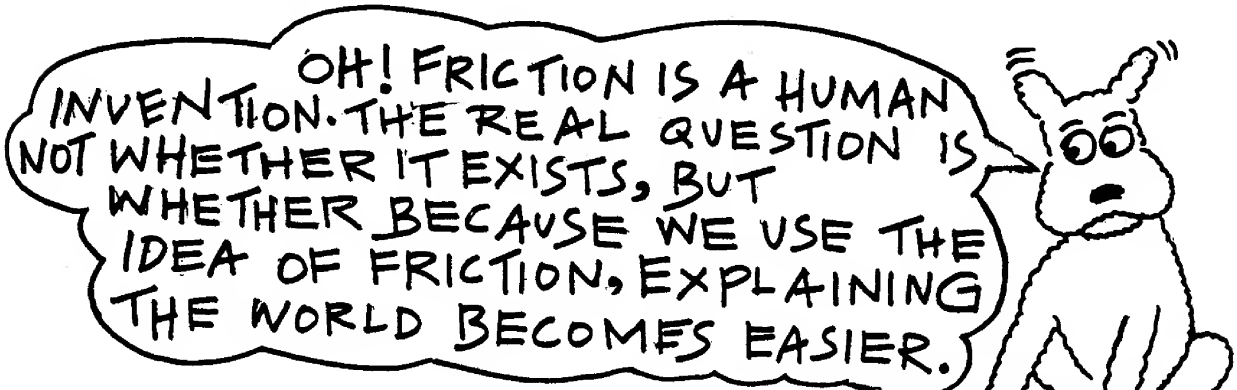
Humans (scientists are humans too!) invent a lot of ideas and concepts - these are ideas in our heads. Real objects do real things. We say they do it because of '**Force**'. Force is our invention. Does it exist out there - this question has no meaning. Force is an idea **we humans** use to explain the real world. It's a tool for our understanding - it does not need a real existence outside in the world. "Is the idea of Force useful" - this is a much better and more real question to ask.

This is an idea that you will face again and again in science - so try to understand it well. To get a real grip on science, you need to understand this idea.

Whether something exists is a simple yes or no question. But when you realize a concept is an invention - that it might be useful - you know you have a choice. You need to know what are the alternative ideas and why this is a better choice. You have to know the reasons for this choice.

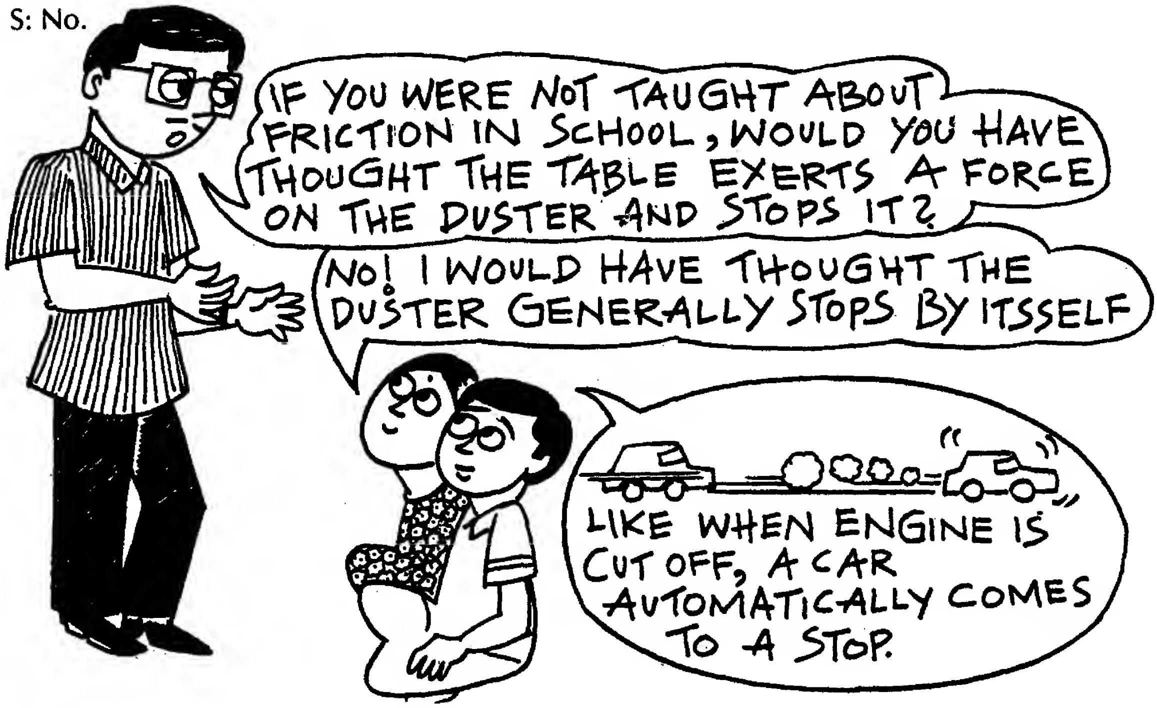
When we say "there is friction" - we are really saying "we choose to use the idea that there is friction because it makes understanding the world easier and simpler." But why did we choose this idea? What were the alternate ideas? How does it make explaining the world simpler? Without looking at all these, you can never really understand "friction" (or such similar concepts) fully.

The concept should make understanding the world easier. I am not saying the concept should be itself easy to understand or accept! Rather, because of the concept, the world should be easier to explain! This is the underlying idea in all of science. We change our ideas to suit the facts we see in the world - we do not change facts to suit our ideas!



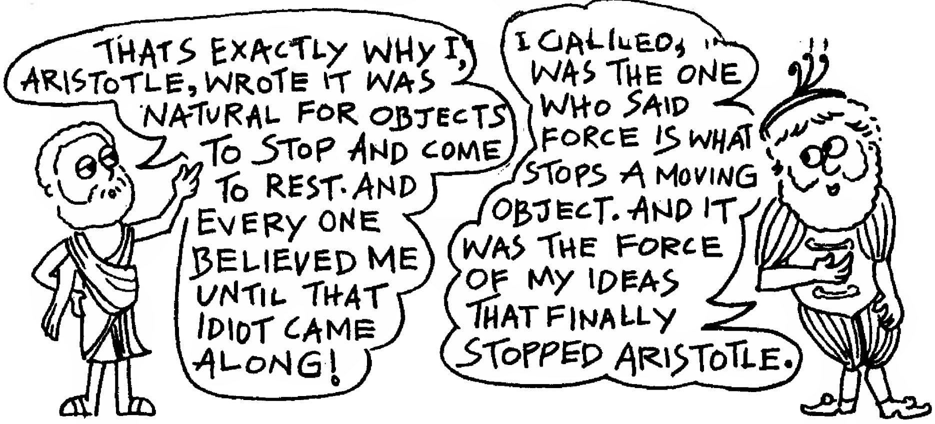
T: Is the there friction or not? This seems like a simple question - and often teachers give a simple answer 'yes'. But here we won't do that. Let's spend some time trying to understand the question and the answer in detail. In reality this is not such a simple question to answer. What do you mean by this question? Can you see friction?

S: No.



T: Exactly. You would have thought stopping was the natural thing to do. When would you be more surprised - if the duster kept on moving for a long time or if it stopped?

S: When it kept on moving. Stopping is what usually happens.



T: You don't see friction. When I push, I exert a force. But the table seems inert - can it exert force? After being taught there is friction, one maybe able to say the table exerts friction force. But what about the first fellow who came up with this idea? How did he/she conclude there was friction?

T: This is a tricky question - you have to think hard about it. Why did Galileo disagree with Aristotle and many others before him? What made Galileo disagree? How did he convince the rest?


ALL OBJECTS COME TO REST IF THERE IS NO FORCE.
NO, IN DEEP SPACE, IF AN OBJECT IS PUSHED, IT WILL KEEP ON MOVING FOREVER.
WHAT NONSENSE IT WILL COME TO A STOP.
NO. IT WILL KEEP MOVING IN DEEP SPACE.
THIS FIGHT CAN KEEP GOING ON. THE REAL QUESTION IS `HOW DO YOU KNOW'? YOU CANNOT GO THERE AND TEST THIS OUT. AT LEAST NOT IN GALILEO'S TIME. AND I, ARISTOTLE SAY... SEE, IF I PUSH AN OBJECT, IT MOVES AND THEN STOPS.
THAT'S BECAUSE OF FRICTION.
WHAT FRICTION? THERE IS NOTHING LIKE THAT IN THIS WORLD.
WOW! I CAN SEE A GOOD FIGHT STARTING UP!

On Knowledge...

Two Chinese philosophers Chuang-Tze and Hui Tse were arguing...

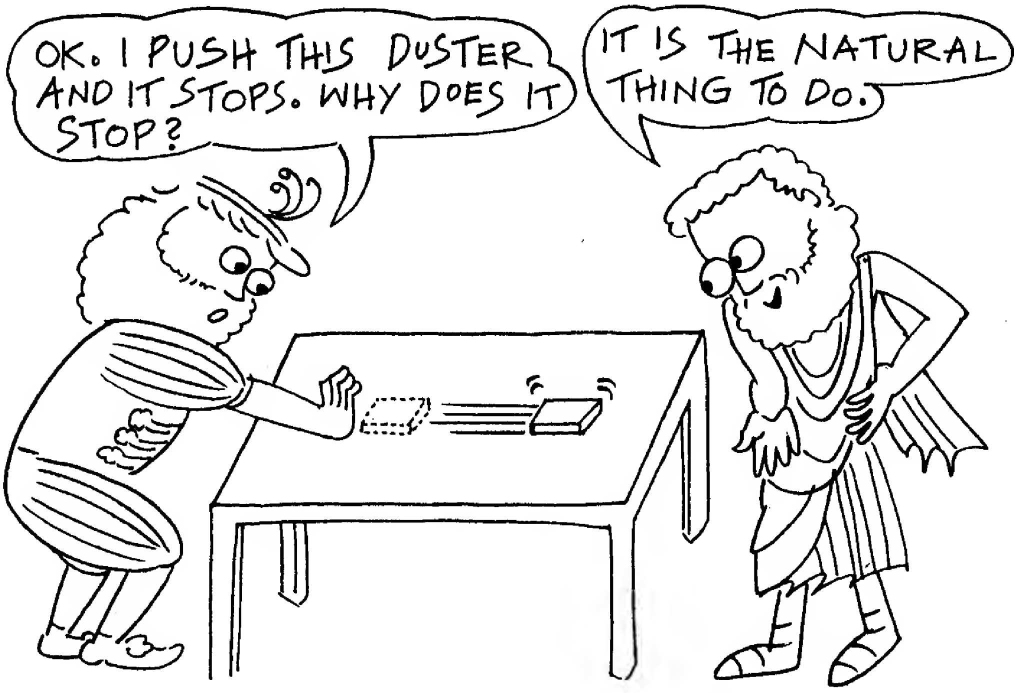
Chuang:	I know.
Hui:	No, you don't!
Chuang:	How do you know that I don't know?
Hui:	How do you know that I don't know that you don't know?

T: This line won't work. You start by assuming there is friction and try to explain things. But Aristotle starts by assuming there is no friction - things just stop. You cannot convince him by repeating the same statement many times. You are not a teacher and he is not a school student! Repeating statements again and again will not convince him. You need to use logic.

What Aristotle is saying is easier to believe. When you change your belief to Galileo's you should have enough evidence and reason for making the change. Galileo really had to think hard and develop arguments and experiments to show he was right. That is the spirit of science. Doing science lies not in the facts themselves - it lies in how you get to the facts.


SO YOU THINK EVERYTHING MOVING COMES TO A STOP?
YES
WHAT ABOUT PLANETS? THEY HAVE BEEN MOVING FOREVER, HAVEN'T THEY?
OOPS! YES, THAT LOOKS LIKE AN EXAMPLE OF MOVING FOREVER.
YOU ARE A BAD ARISTOTLE! THE REAL ARISTOTLE WOULDN'T BE CONVINCED SO EASILY. HE WOULD HAVE SAID "HEAVENLY OBJECTS HAVE DIFFERENT RULES. ANY WAY THEY MOVE IN CIRCLES AND NOT STRAIGHT LINES AND THERE ARE INVISIBLE ANGELS PUSHING PLANETS AROUND. SO WE CAN'T COUNT PLANETS!"

Galileo: I disagree about different laws on earth and in space (heavens). But that is hard to convince you on - since it is just my belief versus yours. So I will skip this argument and try to explain my position from earthly experiments.



Galileo: Ok. But it moves for sometime even after I stop pushing. Why does it do that?

Aristotle: That's because it has some speed and it takes time for this speed to come down to zero.

Galileo: So the speed makes it go some distance before stopping. But how far will it go and stop? Who decides? Is there a rule for this or will the duster decide how far it should go based on its mood?

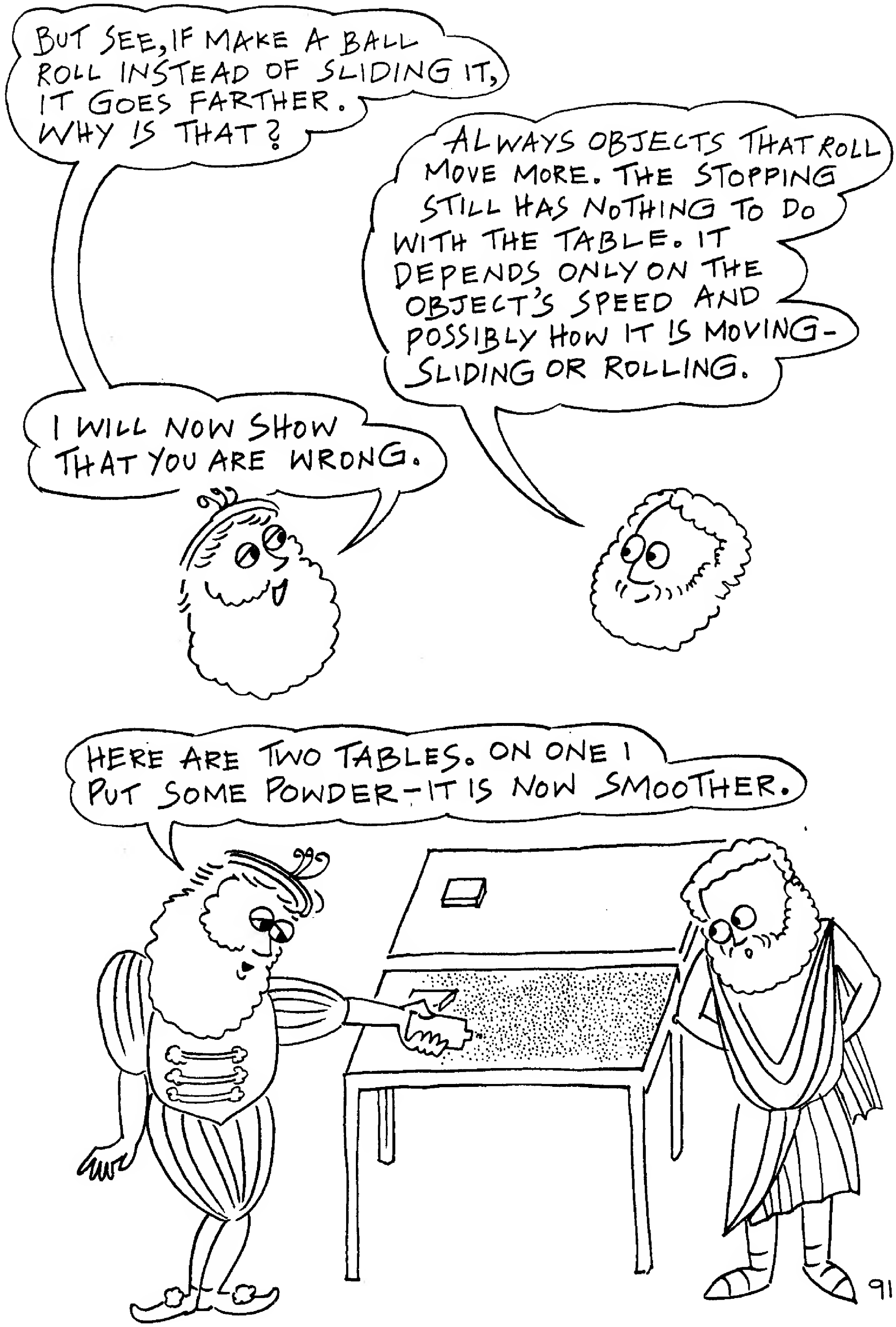
Aristotle: Of course it cannot be random. The distance will be decided by the speed. A faster object will take longer to come to rest and so it will go a farther distance.

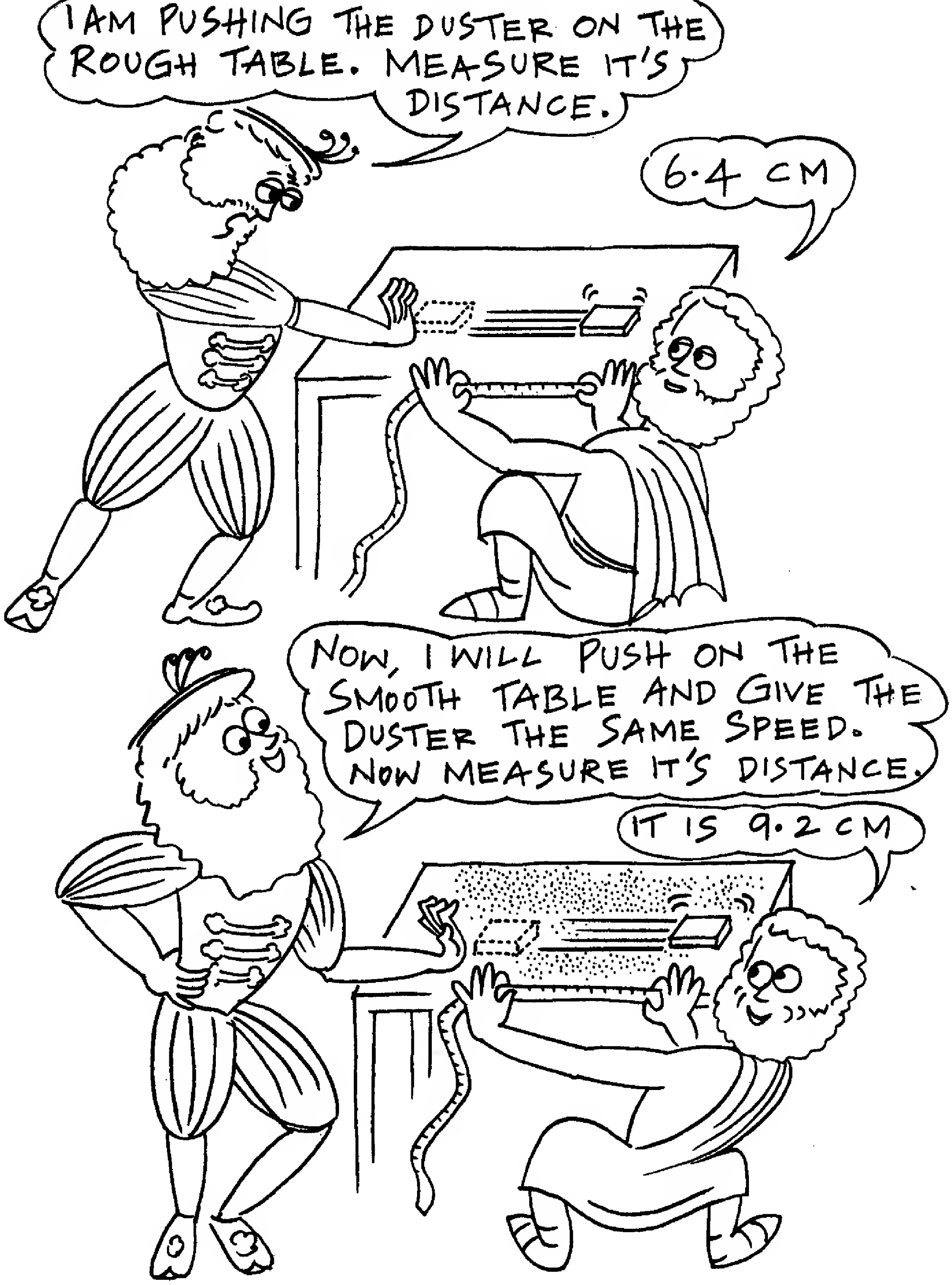
Galileo: And you say the table has nothing to do with this stopping or slowing the duster. No friction, nothing ?

Aristotle: Absolutely nothing.

Galileo: So I presume the table also has nothing to with how far the duster moves before stopping.

Aristotle: Yes.

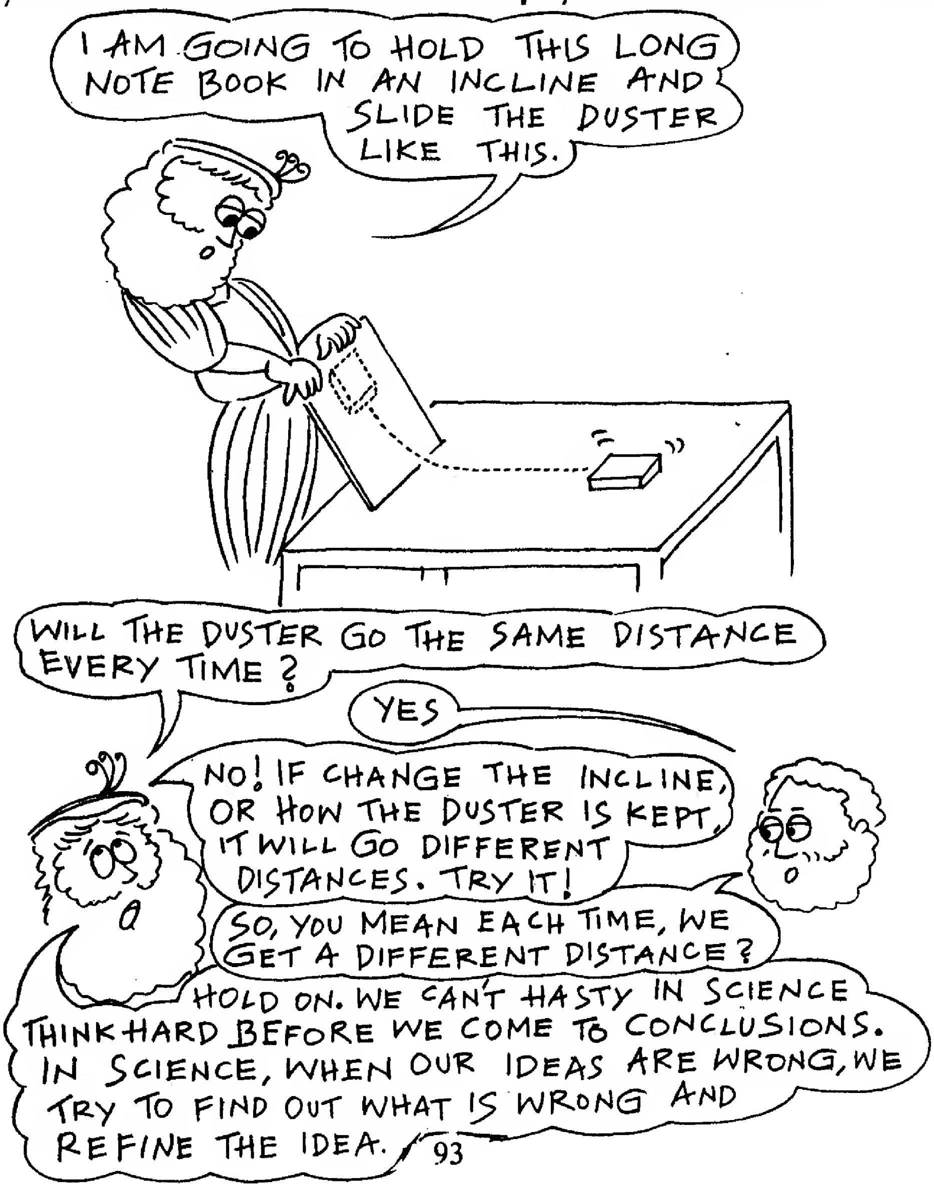

BUT SEE, IF MAKE A BALL ROLL INSTEAD OF SLIDING IT, IT GOES FARTHER. WHY IS THAT?
ALWAYS OBJECTS THAT ROLL MOVE MORE. THE STOPPING STILL HAS NOTHING TO DO WITH THE TABLE. IT DEPENDS ONLY ON THE OBJECT'S SPEED AND POSSIBLY HOW IT IS MOVING - SLIDING OR ROLLING.
I WILL NOW SHOW THAT YOU ARE WRONG.
HERE ARE TWO TABLES. ON ONE I PUT SOME POWDER - IT IS NOW SMOOTHER.


I AM PUSHING THE DUSTER ON THE ROUGH TABLE. MEASURE IT'S DISTANCE.
6.4 CM
NOW, I WILL PUSH ON THE SMOOTH TABLE AND GIVE THE DUSTER THE SAME SPEED. NOW MEASURE IT'S DISTANCE.
IT IS 9.2 CM

Galileo: With the same speed, the same duster moves different distances on the two tables. Clearly the distance moved must have something to do with the table. Isn't it our experience that on smoother surfaces objects slide for longer?

Aristotle: Yes. I guess I have to agree with you. On smooth surfaces objects slide for longer. But how does that mean an object continues moving forever? Also I have an objection to this experiment - you said you gave the same speed to the duster both times. How do you know? In fact I think you cheated and pushed harder the second time.

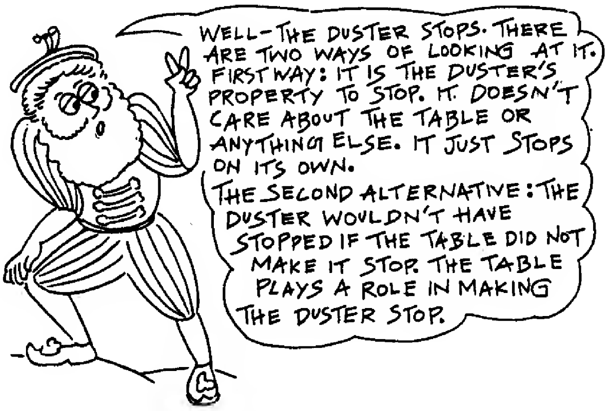
Galileo: Yes - I agree with your objection. I can't be sure that I gave the same speed both times. But I will do this experiment in a slightly different way and then you have to agree with me that I gave the same speed. I will do this experiment next. But before that let's summarize the point you have conceded - **surface smoothness plays a role in the distance moved**.




LET'S KEEP THE NOTE BOOK AT THE SAME INCLINE ALWAYS. IT IS HARD BUT DO-ABLE! AND I WILL LEAVE THE DUSTER THE SAME WAY, FROM THE SAME POINT. LET'S SEE WHAT HAPPENS?
AFTER 4-5 ROUNDS OF THE EXPERIMENT...
SO, WHAT HAPPENED
IT MOVED ALMOST THE SAME DISTANCE EVERY TIME.
YES. THERE ARE MORE FACTORS-THE FAN, YOUR BREATHING, MY HAND SHAK--ING... BUT THE IDEA IS CLEAR THE DISTANCE IS ALMOST THE SAME ALWAYS. IF EVERYTHING IS KEPT THE SAME, IT WILL MOVE THE SAME DISTANCE ALWAYS!
THE FIRST TIME, I KNOW IT MOVES TILL HERE
THE SECOND TIME, I PUT POWDER ON THE TABLE, BUT NOT ON THE NOTE BOOK. THE DUSTER COMES DOWN WITH THE SAME SPEED BUT IT GOES FURTHER ON THE SMOOTHER POWDERED TABLE.

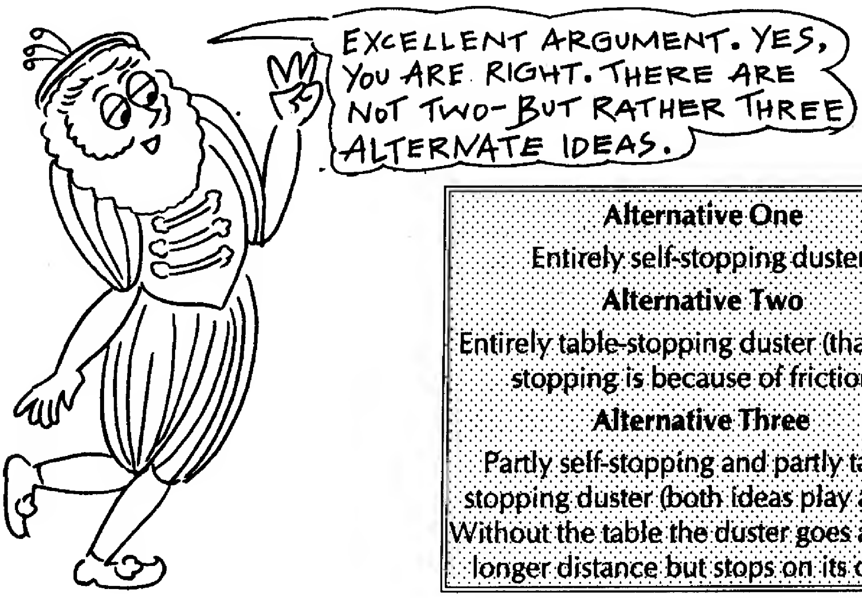
Galileo: So Dear Aristotle, do you agree that we have the same speed and different distances?

Aristotle: Yes. But on this I was convinced anyway. I know on a smoother surface the object moves for a longer distance. But I don't see why that means there is this thing you call friction.



Galileo: We know through experiments above that the table plays a role. Otherwise how can you explain the duster moving a different distance on the smoother table? So the table must be doing something to the duster. It must be exerting a force to stop it. This force is friction. Otherwise the duster would have kept moving forever.

Aristotle: Wait a minute. Ok. I agree that the table plays a role. I even agree that the table may be exerting a force - friction or whatever you want to call it. But why should there be only the two options you have given? Maybe the duster also wants to stop and the table also makes it stop earlier ? So if there was no table, the duster will stop after a longer distance.



Alternative One

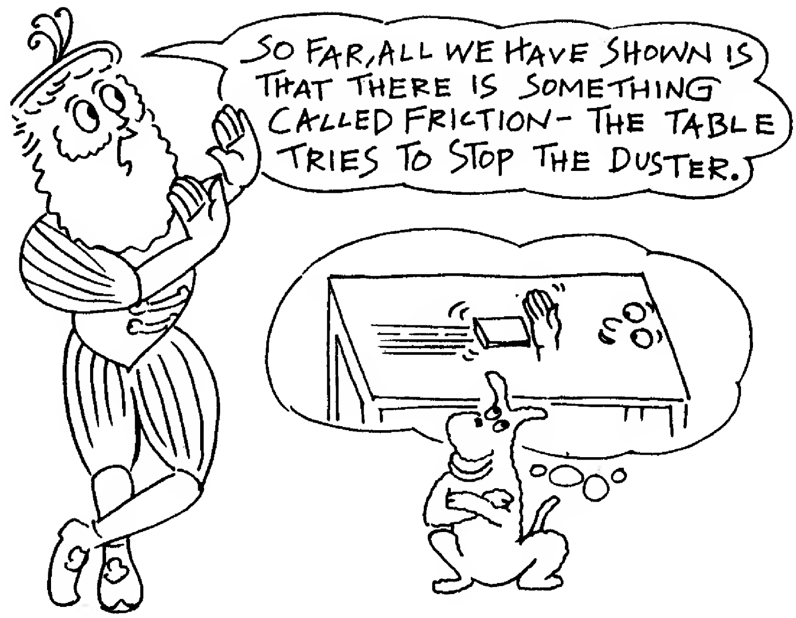
Entirely self-stopping duster.

Alternative Two

Entirely table-stopping duster (that is, all stopping is because of friction).

Alternative Three

Partly self-stopping and partly table-stopping duster (both ideas play a role. Without the table the duster goes a much longer distance but stops on its own).



Galileo: The rougher the table, the more force with which it tries to stop. We have proved that **Alternative One is wrong.** But this does not mean the duster cannot stop on its own. We have not proved Alternative two is correct. Maybe Alternative Three is the correct one. For this I will do one more experiment. But before that let me summarize the points I have won!

Summary so far...

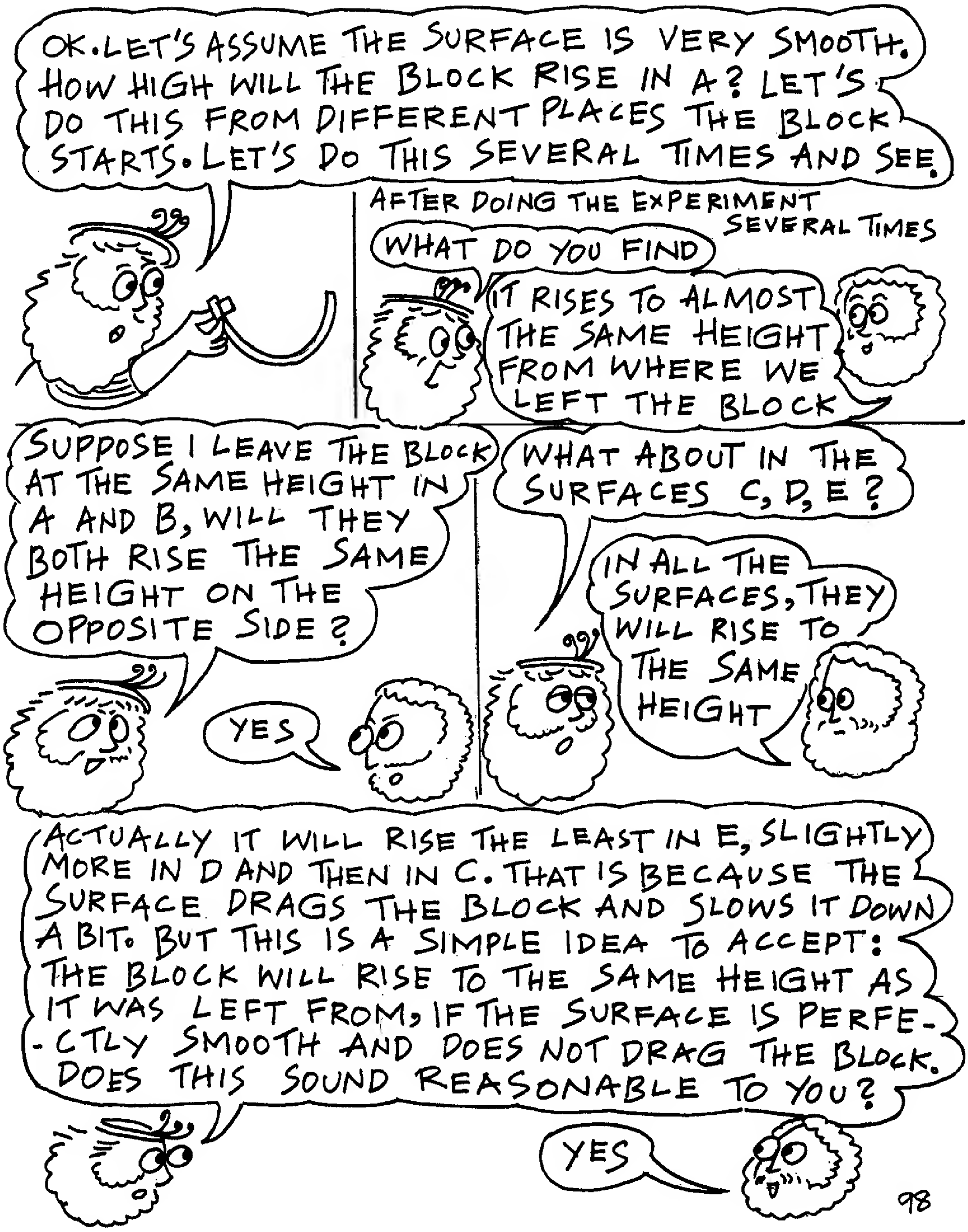
*Same duster speed + same table => **Same** stopping distance.*
*But, same duster speed + different table surfaces => **Different** stopping distances.*

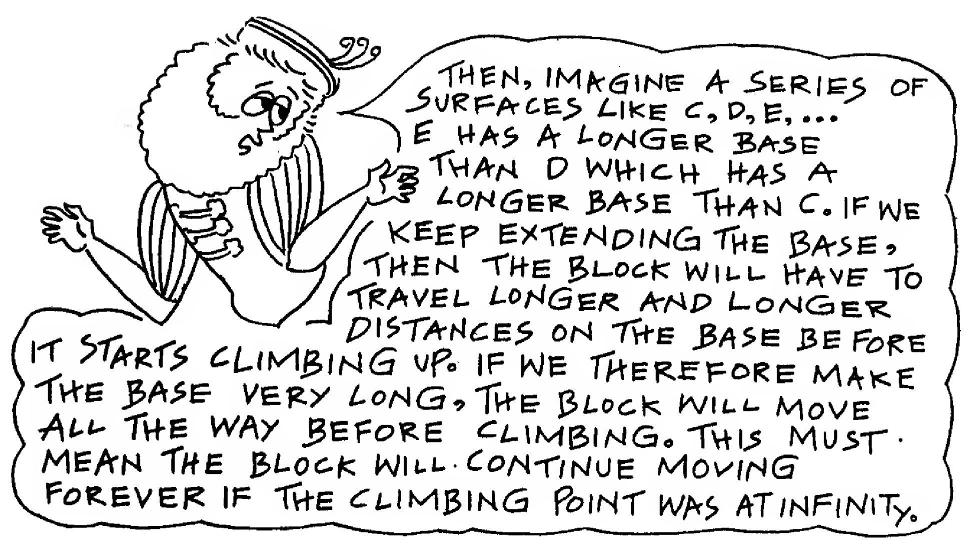
Duster's stopping is not independent of the table. Table matters.
Table tries to stop the duster. The rougher the table, the more it tries to stop.

Alternative one is definitely wrong! Either Alternative two or three is correct.

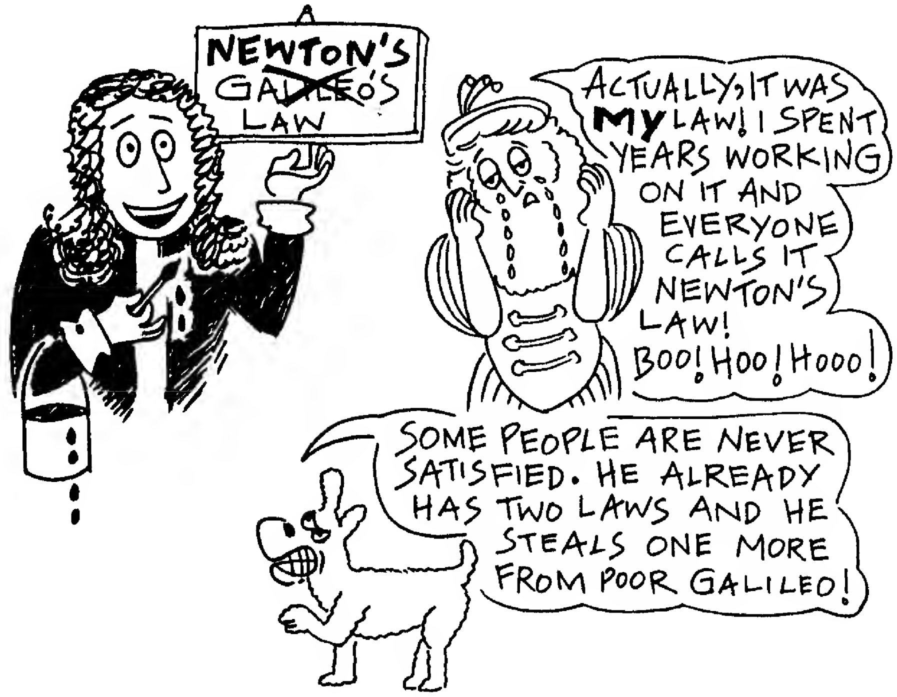
T: Nothing of what we have done so far is very unexpected or surprising. It is all very believable. The only difference is that we have now thought deeply and have done experiments and come up with concrete justifiable statements. This logical *thinking through* will now help us jump to something which is quite surprising. We have so far seen that the table plays a role in stopping the duster. But we cannot say the duster cannot also stop on its own. The next step is to show that the duster actually does not stop on its own. We will show that the entire stopping is done by the table. If the table does not stop it, the duster will keep moving forever.

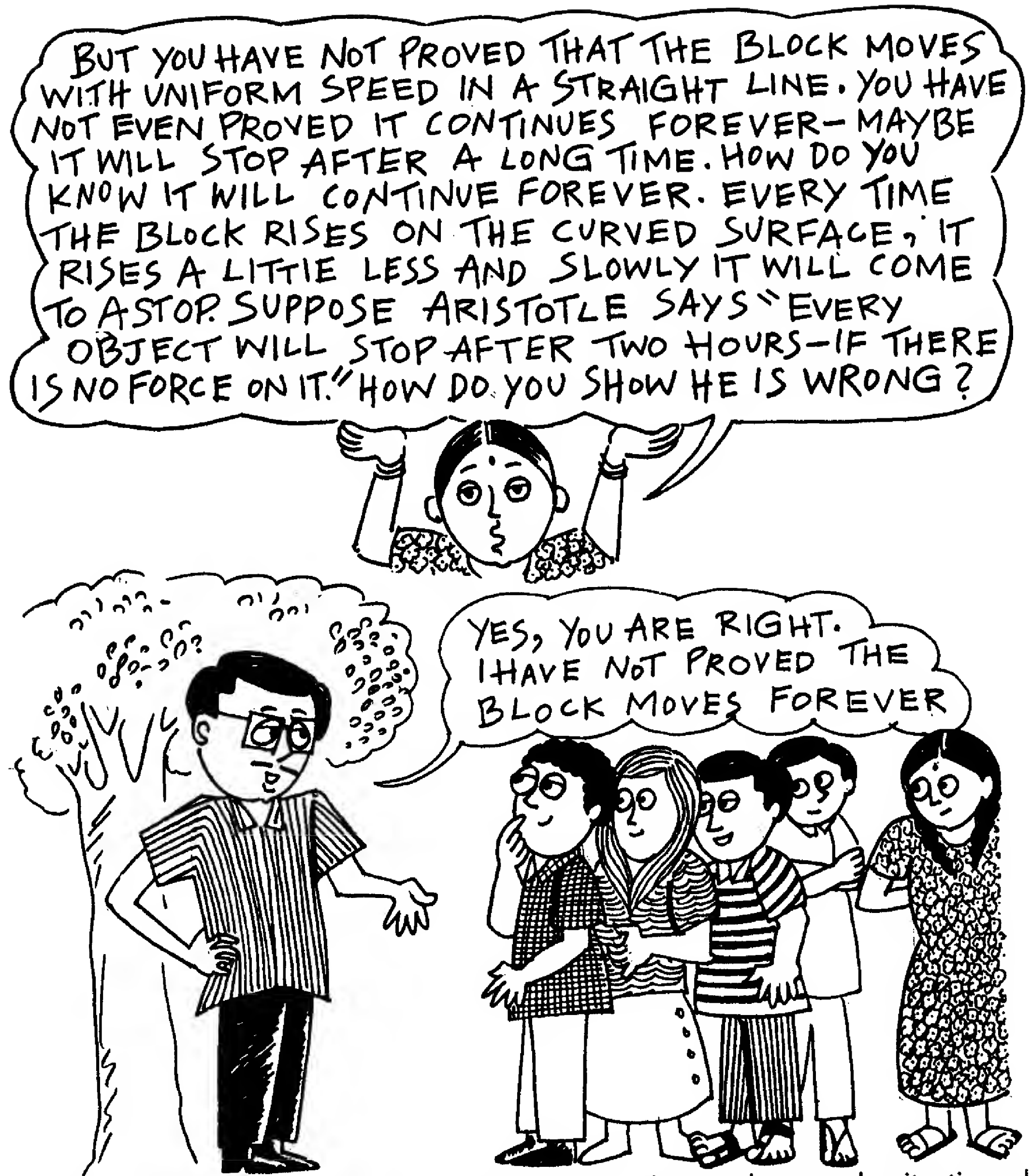

NOW, WE DID THE EARLIER EXPERIMENT WITH THE INCLINED NOTE BOOK. NOW I WILL USE SEVERAL VERY SMOOTH CURVED SURFACES LIKE THE ONES BELOW...
A
B
C
D
E
I RELEASE THE BLOCK FROM THE POSITION SHOWN IN THE SURFACE A. WHAT WILL HAPPEN?
THE BLOCK WILL GO DOWN.
WILL IT THEN STOP?
NO, IT WILL GO UP, STOP AND THEN COME DOWN AGAIN AND GO UP ON THIS SIDE AND KEEP OSCILLATING FOR SOME TIME AND THEN STOP.
IF THE SURFACE IS ROUGH, THEN WHAT WILL HAPPEN?
IT WILL DO LESS OSCILLATION. OR IT CAN EVEN JUST COME DOWN AND STOP. SOME TIMES IT CAN EVEN JUST STAY THERE WITHOUT COMING DOWN.


OK. LET'S ASSUME THE SURFACE IS VERY SMOOTH. HOW HIGH WILL THE BLOCK RISE IN A? LET'S DO THIS FROM DIFFERENT PLACES THE BLOCK STARTS. LET'S DO THIS SEVERAL TIMES AND SEE.
AFTER DOING THE EXPERIMENT SEVERAL TIMES
WHAT DO YOU FIND
IT RISES TO ALMOST THE SAME HEIGHT FROM WHERE WE LEFT THE BLOCK
SUPPOSE I LEAVE THE BLOCK AT THE SAME HEIGHT IN A AND B, WILL THEY BOTH RISE THE SAME HEIGHT ON THE OPPOSITE SIDE?
YES
WHAT ABOUT IN THE SURFACES C, D, E?
IN ALL THE SURFACES, THEY WILL RISE TO THE SAME HEIGHT
ACTUALLY IT WILL RISE THE LEAST IN E, SLIGHTLY MORE IN D AND THEN IN C. THAT IS BECAUSE THE SURFACE DRAGS THE BLOCK AND SLOWS IT DOWN A BIT. BUT THIS IS A SIMPLE IDEA TO ACCEPT: THE BLOCK WILL RISE TO THE SAME HEIGHT AS IT WAS LEFT FROM, IF THE SURFACE IS PERFE-CTLY SMOOTH AND DOES NOT DRAG THE BLOCK. DOES THIS SOUND REASONABLE TO YOU?
YES



T: Close your eyes and imagine this situation. It is very hard to get a practical feel for this idea. But some thought experiments like this can help you develop a feel for this. This kind of *moving forever* must be true for all blocks - not just this block. Also, we have constructed a special experiment to show this 'moving forever' works. Obviously, you don't need to slide down to keep moving forever. If you are already moving, and nothing stops you, you will naturally keep moving forever. This is Newton's first law. Actually it really was Galileo's law.



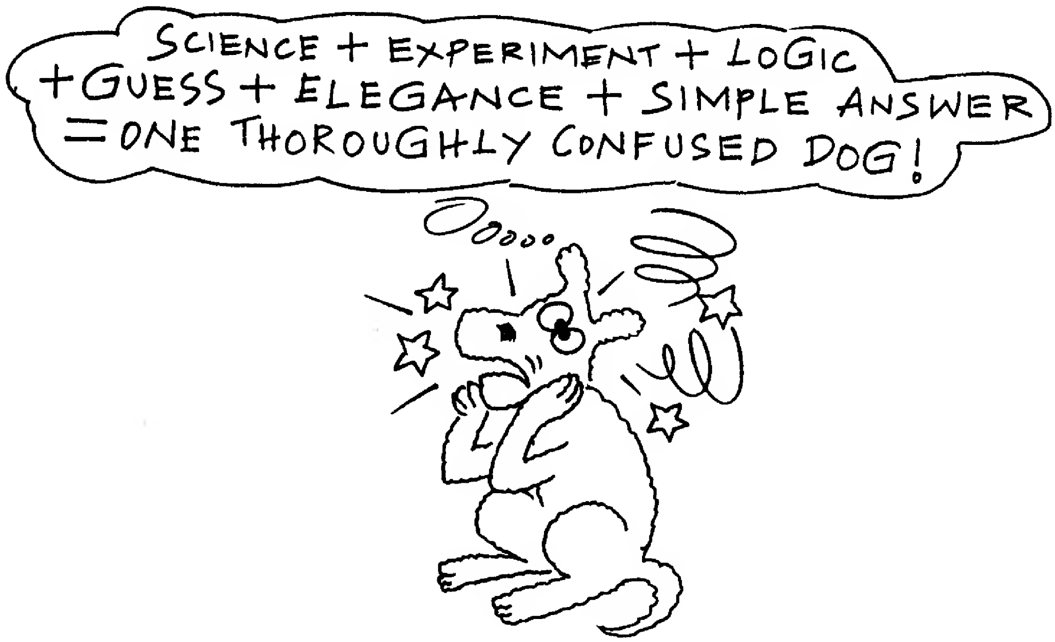


T: I have not proved that the block continues forever. I have only created a situation where you can get a **feel** for why the block **can** keep moving forever. If Aristotle says the object will stop after two hours - then I will devise an experiment to show that objects can move for 3 hours before stopping. It will be hard to show - but today we can do it with some complex equipment. But then Aristotle will come back to say "Oops! I made a mistake. I wanted to say every object will stop after 2 years, not two hours." Now it is much harder to show this is wrong. But let's say we send a spacecraft (like Voyager which has really been going in a line for more than 10 years) into space and then show Aristotle is wrong. Then all Aristotle has to do is to say "Sorry - I meant 200 years."

T: This can go on forever. So we must make rules - Aristotle cannot keep changing his law. Second we have to accept that we cannot **prove** we are right. All we have done so far is to show that the idea that objects can keep moving forever is not very far fetched. Initially this sounded ridiculous - but with a bunch of experiments and arguments we have hopefully made the idea look possible. But this is not proof. **There is no proof.** This law is the starting point. We use this to deduce other facts. If these facts turn out right, then we develop more confidence in the idea.

S: So you are saying we cannot prove Newton's first law? We cannot say Alternative three is wrong - that objects do not stop automatically?

T: We can and do **say** Alternative three is wrong! We only cannot **show** it is wrong. What we know for sure is that there is some role for the table and therefore for friction - Alternative one is wrong. Measurements show that as the table becomes smoother, the rate of slowing down becomes very small. But it is never zero. We cannot show the object does not slow down on it own. We can of course show that if it exists, this self-slowing has to be very small. Science is not all logic and experiments! There is also guesswork and hope for elegant and simple answers. These also drive science.



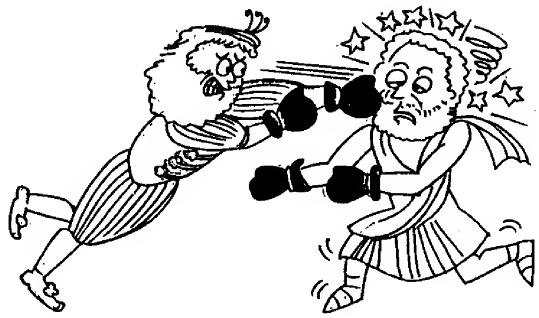
How Science Works ...

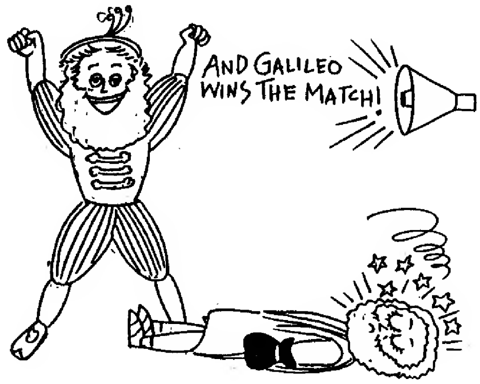
Once we get a feel for something, we generalize. We guess that there is no self-slowing down. If all the slowing down was self-slowing, it would have been simple. If all of it is friction, that is also reasonably simple - it requires us to change our ideas but it is still simple. Alternative Three - some small self-slowing and a lot of friction is much more messier than Alternative One and Two. We cannot say things slow only because of self-slowing. Alternative one - simple to understand - is clearly wrong. So we hope and guess Alternative Two is correct - because, of the remaining two alternatives, it seems simpler and less messier than Alternative three. We build our ideas based on this assumption. Our ideas make predictions that we check with the real world. If it works, then we stick to the assumption. Otherwise we do more checks and change the assumption.

Maybe things do have a built in self-slowing. We do experiments and show this self-slowing takes at least 2 minutes to make things stop. Better experiments show it takes at least 2 hours to make things stop. Even better experiments stretch the limit to 2 years and to 2 centuries. This self slowing has to be very very small. But we cannot show it is zero. In science it is easy to show our ideas are wrong. It is very difficult to show we are right. So we **assume** we are right. If we get into experimental conflict with nature, we will change our ideas. Science is a delicate blend of experiments, logic, and inspired guesswork that comes from a feel for nature. All three things are essential in doing science.

Historically Speaking...

Galileo actually used the above arguments to convince himself and others that objects keep on moving forever. He was able to shift himself and others from the idea that being at rest was the only natural state for an object. He did a lot of experiments to see how far objects move on different smooth surfaces.





He found that the speed decreased slowly. For a given surface the body slowed down at particular fixed rate. As the surface became smoother, he found the rate of decrease decreased. He concluded that the slowing down was caused by friction and if there was no friction, the body will keep moving at the same speed in a straight line.

There were many who were not convinced. But they all died out. The newer generation grew up with Galileo's ideas and found it more useful. Science often wins - not by convincing but by simply waiting long enough!

Since we finally did not prove this law - maybe you feel we could as well havel simply stated the law as the first step and said this is how the world is. There are two problems with this. One is that you don't know why we think this law is right and how much 'evidence' we really have for it. The second is that you don't see the counter-options and the arguments for and against this law - therefore you don't get a complete feel for the 'law'. You learn the 'fact' but not how we arrived at this fact. This long-winded process hopefully has given you a better insight into why we believe in this law. You can similarly question other laws and delve into the experimental and logical basis for these assumptions.

Summary of Points Discussed

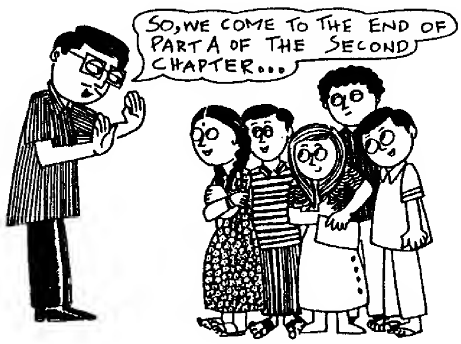
1. Though moving things always stop on the earth, this is not the most natural thing for objects to do.

2. If we assume that naturally moving objects stop, then careful observations (like distance moved before stopping) become harder to explain.

3. A framework where we assume moving things naturally keep moving in a straight line forever is a better framework to work with. It makes it easier to explain our observations. Within this framework, stopping is caused entirely by a force. Often friction between two surfaces plays this role.

4. In situations where friction and other such forces can be eliminated, things will keep moving in a straight line forever. A lot of our current physics is built on this critical hypothesis and we find this contradicts nothing we have met so far - this makes our faith in this law very very strong.

5. So internalize these key ideas
   a. The **Natural Motion** of objects is that of **constant velocity** - moving uniformly forever in a straight line. No force is required to **move** an object. A force is only required to start it moving.
   b. Since force is needed to start things moving, one might feel rest is a superior natural state. But rest is relative. And things like stars and galaxies have been always moving with respect to us - so there is no question of *starting of motion* here. In general, constant velocity (with rest as a special case) is the natural state.



Chapter 2 - Section B

Newton's First Law - Motion of Planets

In the previous section, we looked at why we feel moving objects keep moving forever and how that makes explaining motion of dusters on earth easier to explain. In this section, we will apply this idea to the motion of planets and try to understand how this law explain this as well.

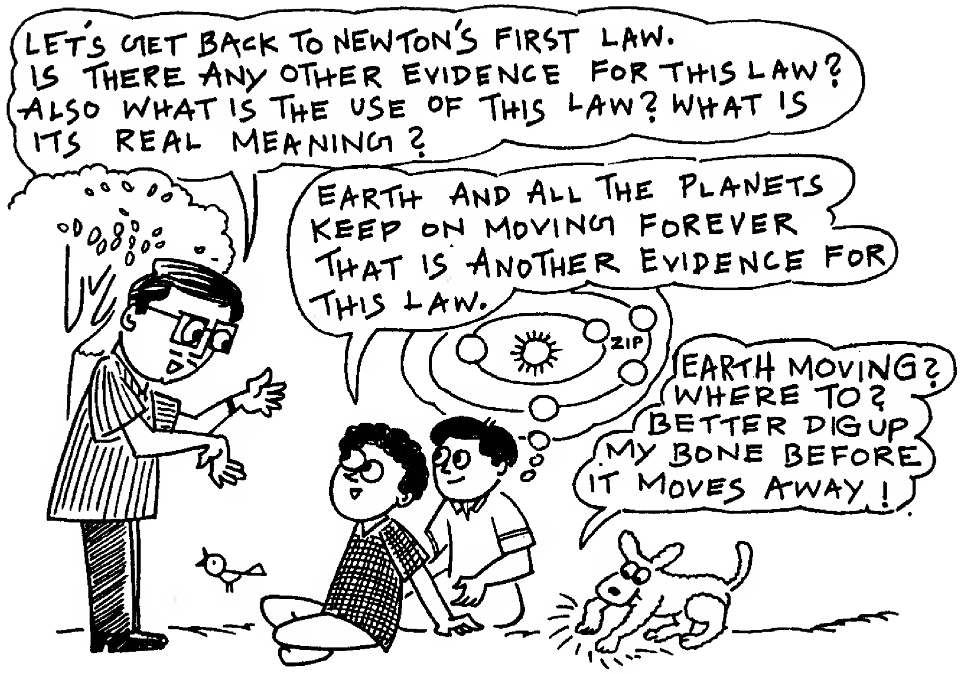
This section starts by looking at the motion of the moon around the earth. Everyone has heard of Newton's law of gravity - the earth pulls things towards itself. This seems self-evident - when we drop things, they fall down. But that is not really the BIG success of Newton's gravitational law. His major success was in showing that the Moon's motion is governed by the same law. In short what Newton is saying is that not merely stones dropped on the earth, but even huge rocks like the Moon that are so for away, fall towards the Earth. And that is because the Earth is pulling them towards it.

Now, this sounds absurd clearly the moon is not falling towards the earth. Or is it? In this section we show how the moon is actually continuously falling towards the earth - and yet it never quite falls down!

Understanding why circular motion is actually just falling makes use of the idea that objects left to themselves naturally tend to keep moving in a straight line forever. Without this idea, we cannot show that the moon is actually falling. And so the earth couldn't be just pulling it. And explaining why the moon goes around would be extremely hard.

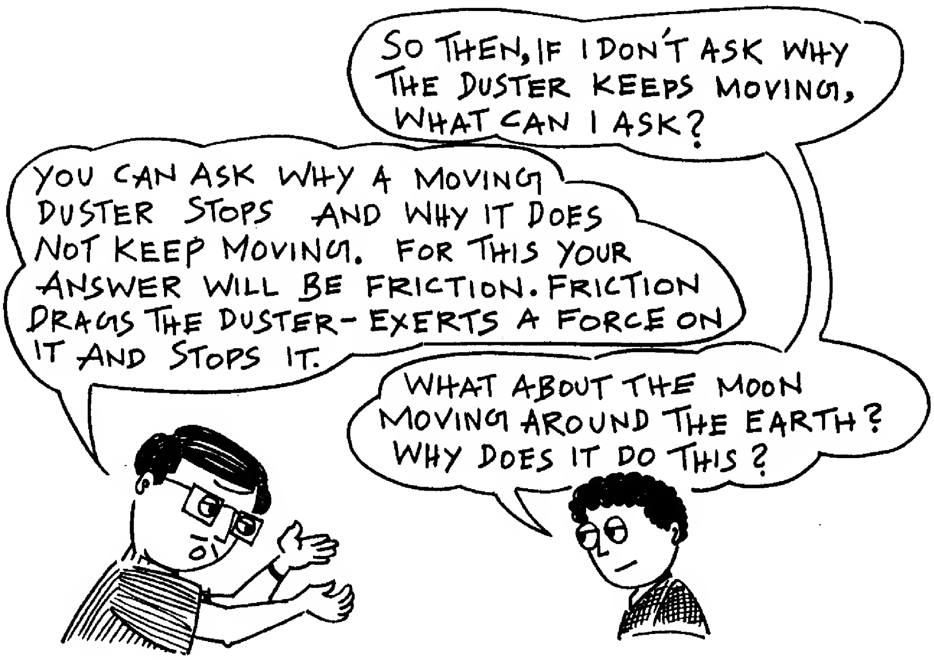
The difference between pre-Newton and post-Newton eras is this - what is natural for an object to do has changed. Earlier people thought of rest as natural. Now not just rest, even uniform motion is seen as natural. This section shows how this seemingly small change can make a huge difference in terms of our ability to explain the motion of objects - heavenly or earthly!

We finally end the section with a historical note on how Newton discovered gravity.

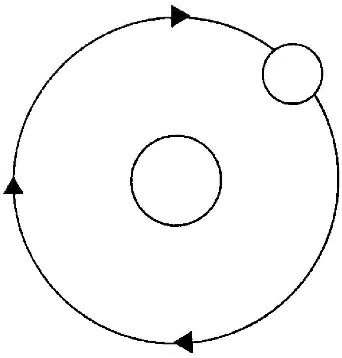


T: Yes - but they do not move in a straight line, nor with a constant speed. But uniform motion forever plays a role in 'planets keeping on moving'. We will try to understand this now. But before we do, let's look at the real meaning of this first law. What the first law says is that you **cannot** ask why something is moving in a straight line with constant speed. That is what it should do naturally.

T: Yes. But duster stopping is not the most natural thing for a duster to do. If it was in space - it would have kept on moving. Sometimes our limited experience on earth teaches us that something is 'normal' and other things are not. But with a larger experience and broader perspective - things that were abnormal appear 'normal' and things earlier 'normal' appear as special situations. Dusters here on earth stop - but that's because our experience is small. If we leave dusters in space - it will keep on moving. To keep moving is natural. To stop is abnormal - though we may expect things to stop because of our limited earthly experience.



T: Earlier people used to ask this same question - unsuccessfully. When you ask a 'why' question - there is always a background to it. Why questions come with an alternative - why is something like this ***and*** not like something else.



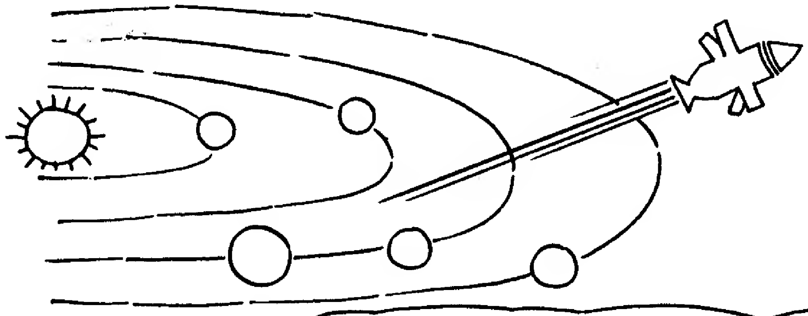
T: This is a hard question - what is it that is making the moon move from rest and making it move around the earth? That something must be pushing it around the earth. See the diagram. If you want the moon to go around and have to push it around from rest, you must be pushing it around the circle (or ellipse). Obviously the earth cannot be doing this job. (Think of yourself in the center and some people running around you in a circle - you can pull and push them towards and away from you - but how do you push them around?)

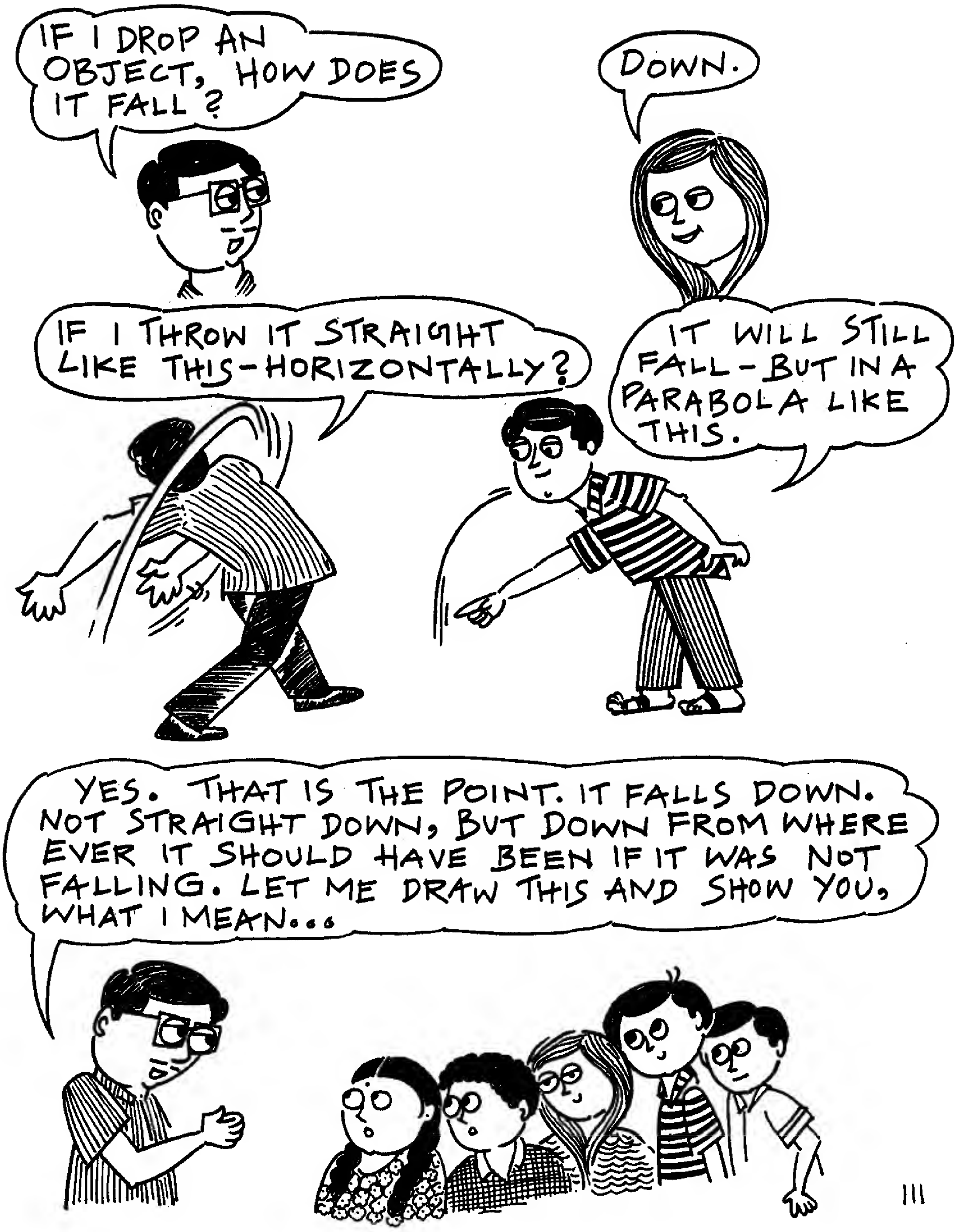
T: I am not saying this argument is perfect. But it sounds somewhat logical. Later Newton proposed that the earth could pull the moon - even when it is so far away, without touching it. One can argue that if the earth can pull things towards it without touching, why can't it push things around in a circle. The argument that forces from you can only be away or towards you is not true. But many of the forces seem to be this way (including our pushing and pulling). At any rate, this was what was argued in the 15th century and this idea that angels sat behind the planets and flapped their wings and pushed them around was quite common. Today we don't buy such arguments. But then how do we explain Moon's going around? Using the idea that moving objects keep on moving in a straight line, Newton showed that moving in a circle is like falling down and that the Earth's pull is enough to explain Moon's going around. Earth's pull is believable because things on earth also fall down. With one step and one simple set of ideas Newton was thus able to explain the motion of things on earth and things in the heavens. This was an important break-through in the history of science. In what follows we will look at the thought process that led to this change.

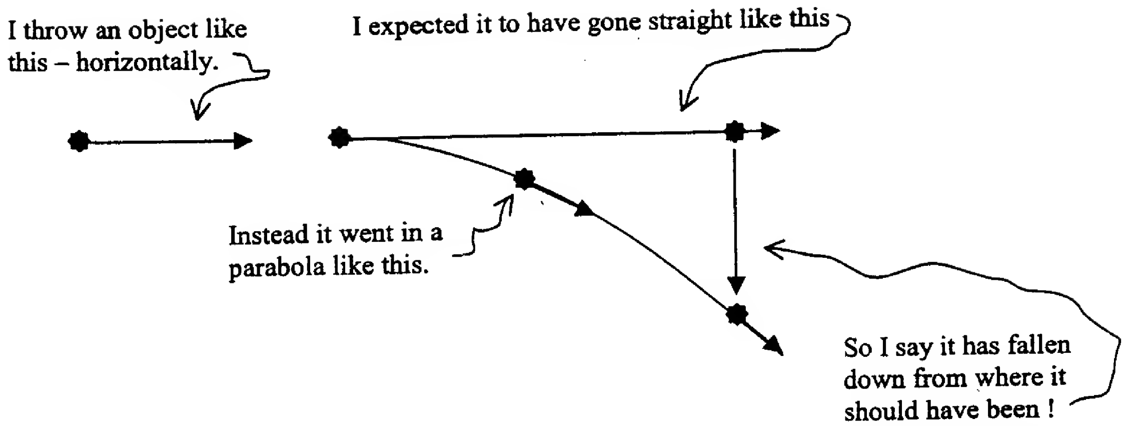
T: But now when you ask why is the Moon going around the earth, you must state what you expect it to do otherwise. This time the answer is not simply "remain at rest there." The natural thing for the Moon is not merely being at rest - it can instead be moving uniformly in a straight line. So when you ask "why is the Moon going around in circles" - you are really asking "why is the Moon **not** going off in a straight line?"


YOU MEAN TO SAY IF THE MOON GOES OFF IN A STRAIGHT LINE, I CANNOT ASK WHY IT DOES THAT?
EXACTLY! IF YOU LEAVE A COIN ON A TABLE AND COME BACK 5 MINUTES LATER AND STILL FIND IT THERE, WILL YOU GENERALLY ASK WHY IS IT STILL THERE?
NO. I EXPECT IT TO BE THERE - IF NO ONE TOUCHES IT.
YES - IT IS NATURAL FOR THINGS AT REST TO REMAIN AT REST. BUT THAT IS ONLY PART OF THE STORY - IT IS SIMILARLY ALSO NATURAL FOR THINGS MOVING TO KEEP ON MOVING IN A STRAIGHT LINE. THIS IS HARD TO INTERNALIZE — BUT THAT IS WHAT YOU HAVE TO DO. ONCE SPACE SHIPS IN SPACE MOVE AWAY FROM THE EARTH, THEY USUALLY KEEP ON MOVING FOREVER - WITH ABSOLUTELY NO FUEL, NO PUSHING, ETC.

T: For example in the 1970s the Voyager space ship left the earth and kept moving on its own after some time. It passed Jupiter and Saturn, crossed Pluto and is now traveling beyond the Solar System. It will keep traveling forever in a straight line till it hits or meets some other planet, star or space rock. Newton's first law works! You cannot ask why the spaceship is moving in a straight line - that is what it is supposed to do. If it stopped, or bounced off etc, you can ask why it did that and you will find reasons. But there is no 'why' to moving in a straight line - that is what it is naturally supposed to do!




IF I DROP AN OBJECT, HOW DOES IT FALL ?
DOWN.
IF I THROW IT STRAIGHT LIKE THIS-HORIZONTALLY?
IT WILL STILL FALL - BUT IN A PARABOLA LIKE THIS.
YES. THAT IS THE POINT. IT FALLS DOWN. NOT STRAIGHT DOWN, BUT DOWN FROM WHERE EVER IT SHOULD HAVE BEEN IF IT WAS NOT FALLING. LET ME DRAW THIS AND SHOW YOU, WHAT I MEAN...

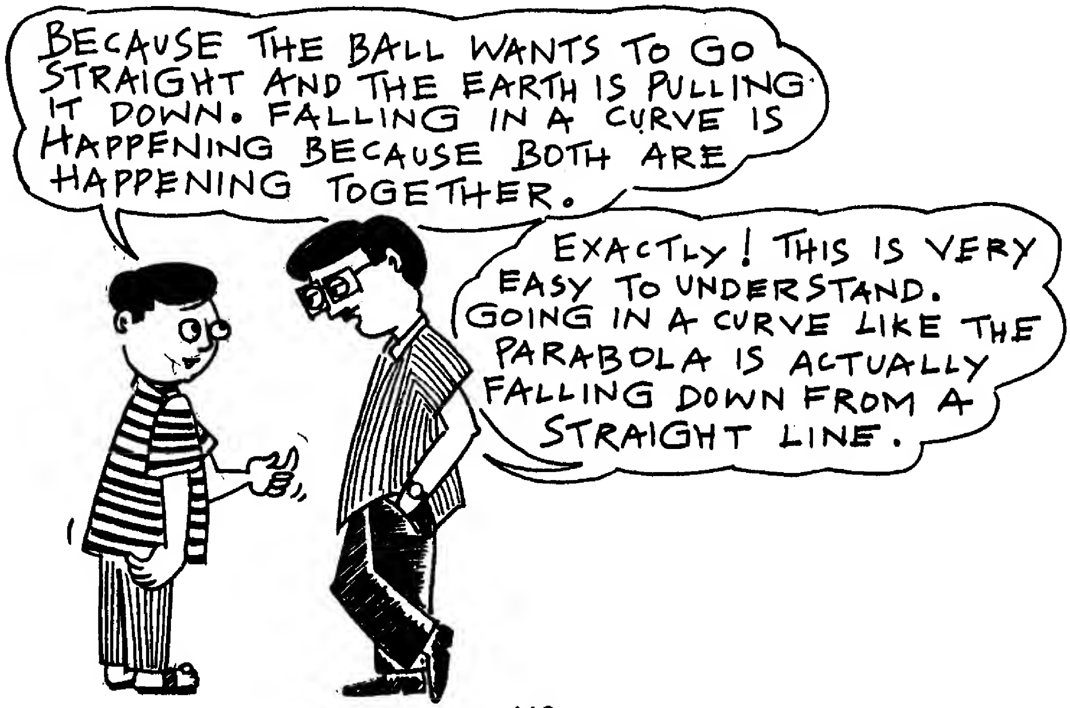


T: Falling down is always from where you otherwise expected it to be. Usually people think of falling down as coming down vertically. But that is not always the case - when moving things fall (like a ball thrown horizontally for example), we talk of falling down as falling from the straight line it should have otherwise gone in.

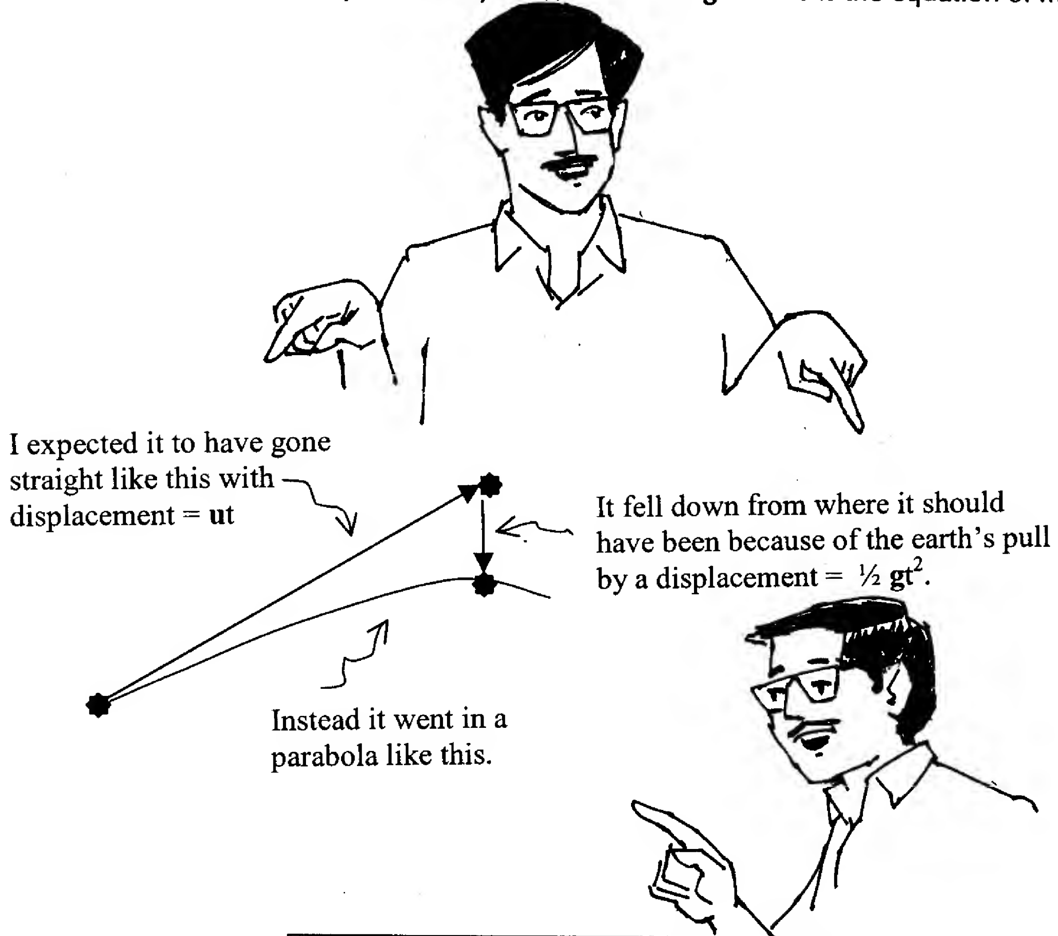
T: Why does the ball fall down from its straight line ?

S: Because the Earth is pulling it down due to gravity.

T: Yes. But the earth only pulls it downwards. Why is it going in a curve like that ?

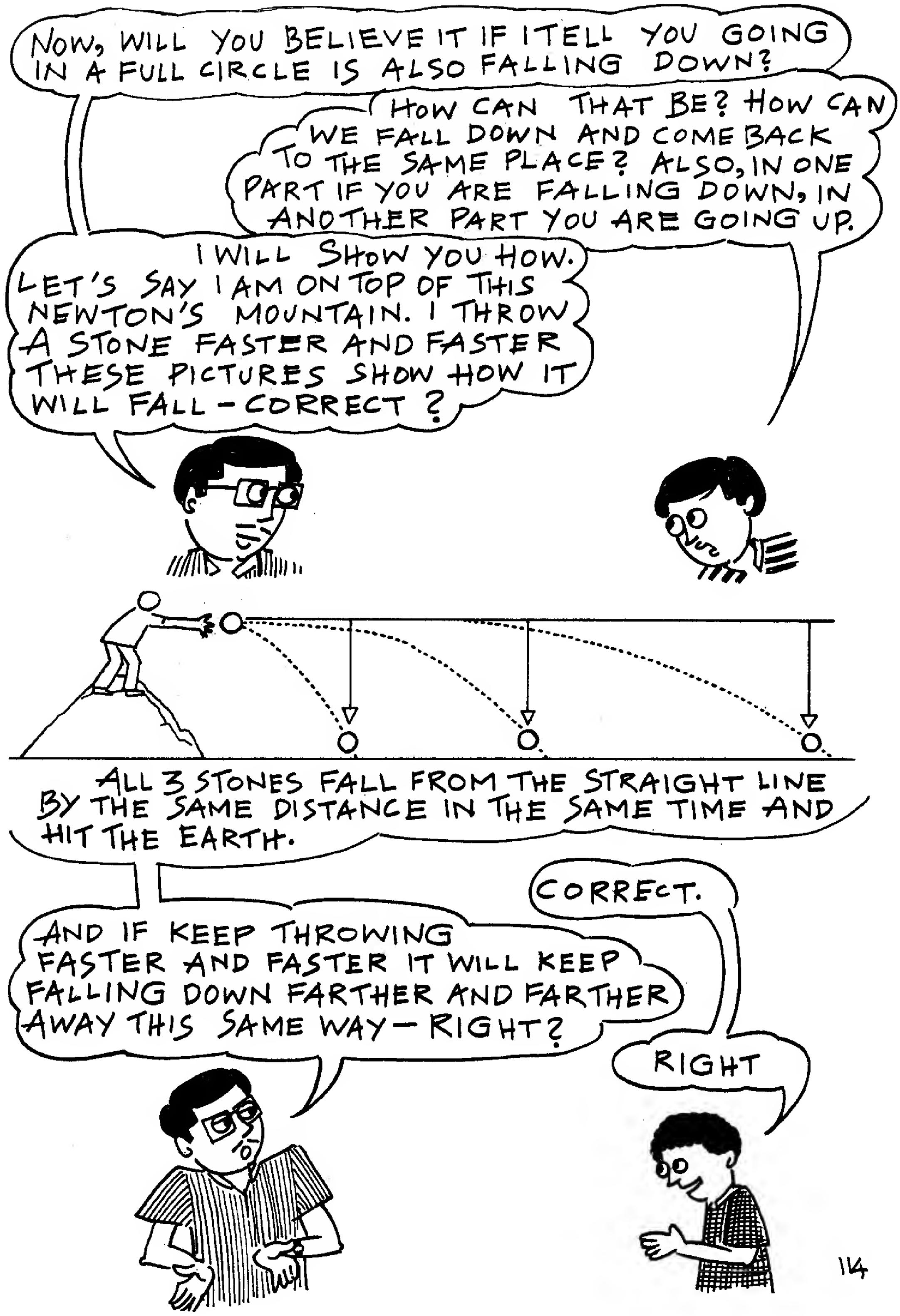


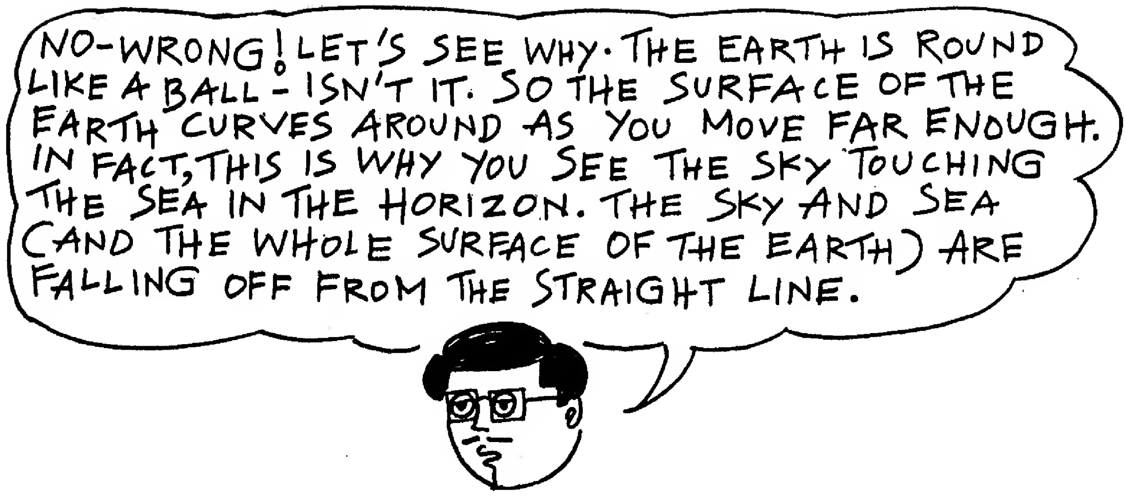
T: In fact for those of you who know the equations of projectile motion - you can think of it like this. As shown in this figure below - the object should be ideally at **ut** (vector **u** is the initial velocity) because it should have gone in a straight line. But earth's pull makes it fall down by ½ $\mathbf{g}t^2$. So the object is really at $\mathbf{r}(t) = \mathbf{u}t + \frac{1}{2}\mathbf{g}t^2$. This is the equation of motion.

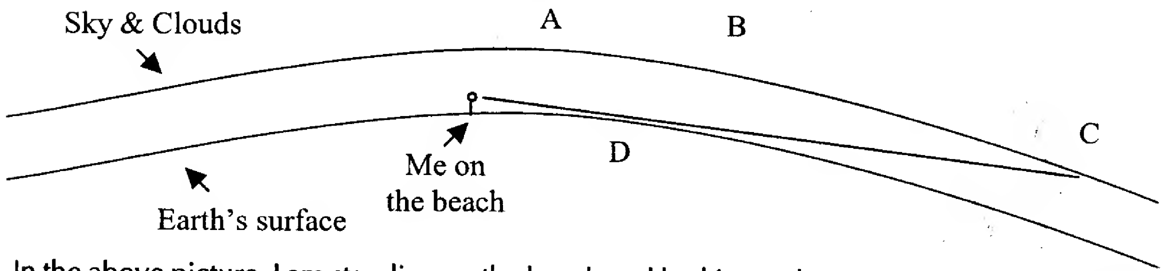


Note that although the ball is 'rising' in the figure - we still talk of it as falling. Falling from where it should have been. This is the critical point you have to understand.

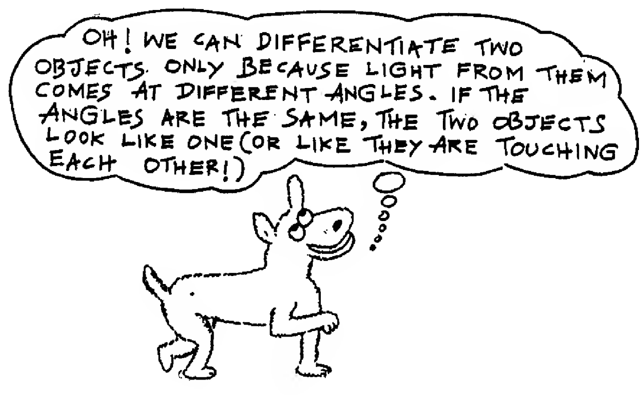
For those who hate equations - I promise no more equations in this chapter. This was the first and the last. If you did not follow the equations don't bother. All you need to understand is that going in a curve is still just like falling down (falling down from a straight line).

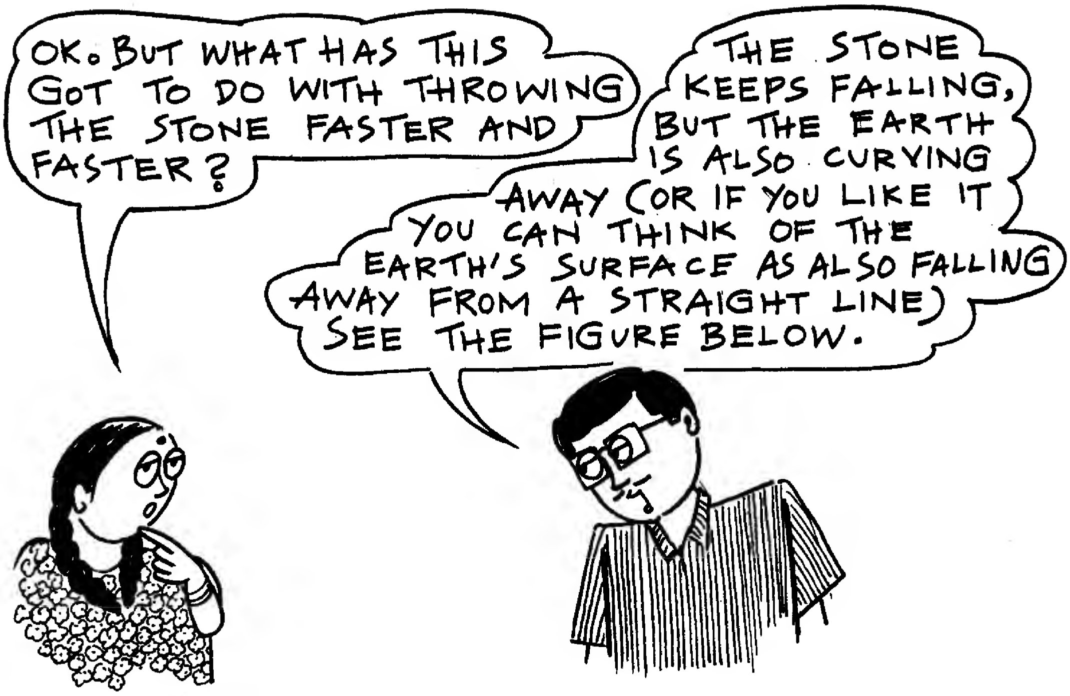

NOW, WILL YOU BELIEVE IT IF I TELL YOU GOING IN A FULL CIRCLE IS ALSO FALLING DOWN?
HOW CAN THAT BE? HOW CAN WE FALL DOWN AND COME BACK TO THE SAME PLACE? ALSO, IN ONE PART IF YOU ARE FALLING DOWN, IN ANOTHER PART YOU ARE GOING UP.
I WILL SHOW YOU HOW. LET'S SAY I AM ON TOP OF THIS NEWTON'S MOUNTAIN. I THROW A STONE FASTER AND FASTER THESE PICTURES SHOW HOW IT WILL FALL - CORRECT ?
ALL 3 STONES FALL FROM THE STRAIGHT LINE BY THE SAME DISTANCE IN THE SAME TIME AND HIT THE EARTH.
CORRECT.
AND IF KEEP THROWING FASTER AND FASTER IT WILL KEEP FALLING DOWN FARTHER AND FARTHER AWAY THIS SAME WAY - RIGHT?
RIGHT

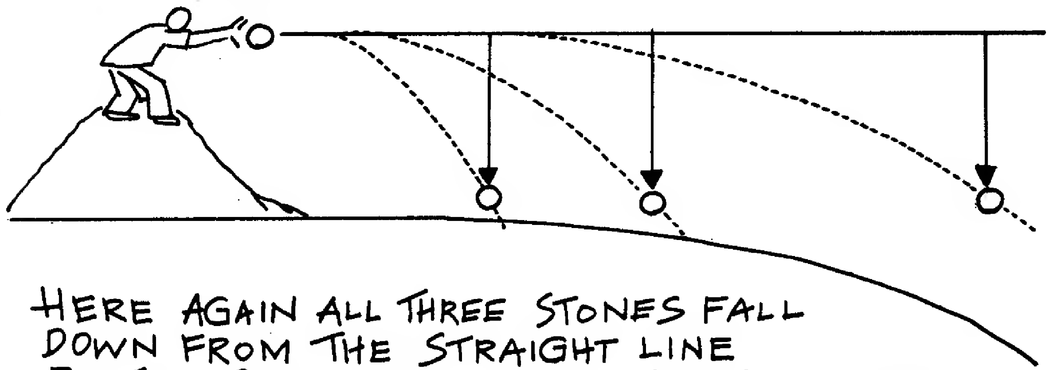




In the above picture, I am standing on the beach and looking at the sea. At position A, the sky is above me and the sea is below me - I can see both when I see up and look down. I can see the Sea till position D. Beyond this the earth's curvature makes the sea go below what my eye can see. Points beyond D on the surface I cannot see because the light from them cannot reach my eye - they are blocked by the earth's surface. But at D, I can still see the sky above me. At position B, I can see the sky, but not the earth or the sea. I can see the sky till position C. Beyond C, I can't even see the sky or clouds. Though C is so far away from D - light from the sky at C and the sea at D both come to me from the same angle. So I think the sky is touching the sea. Actually it is only the light from the sky at C that is touching the sea at D and reaching my eye!

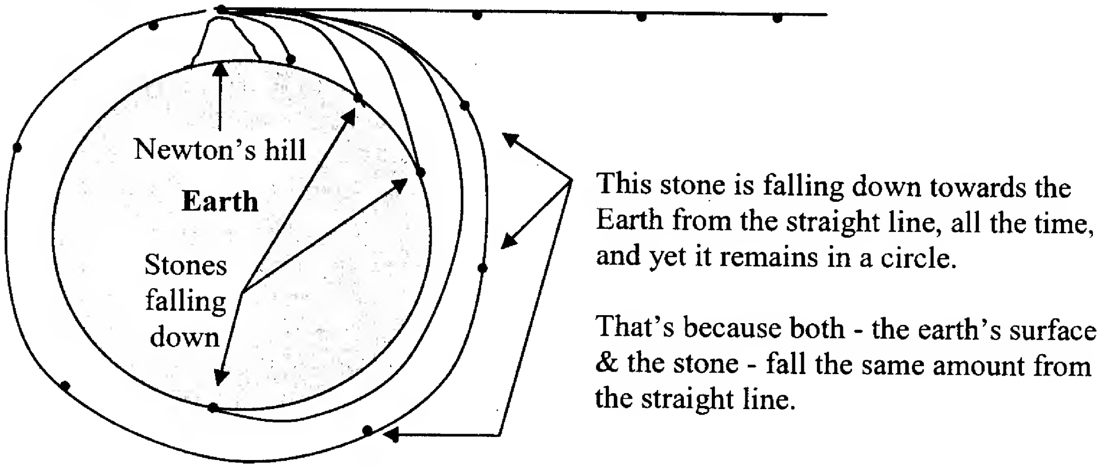




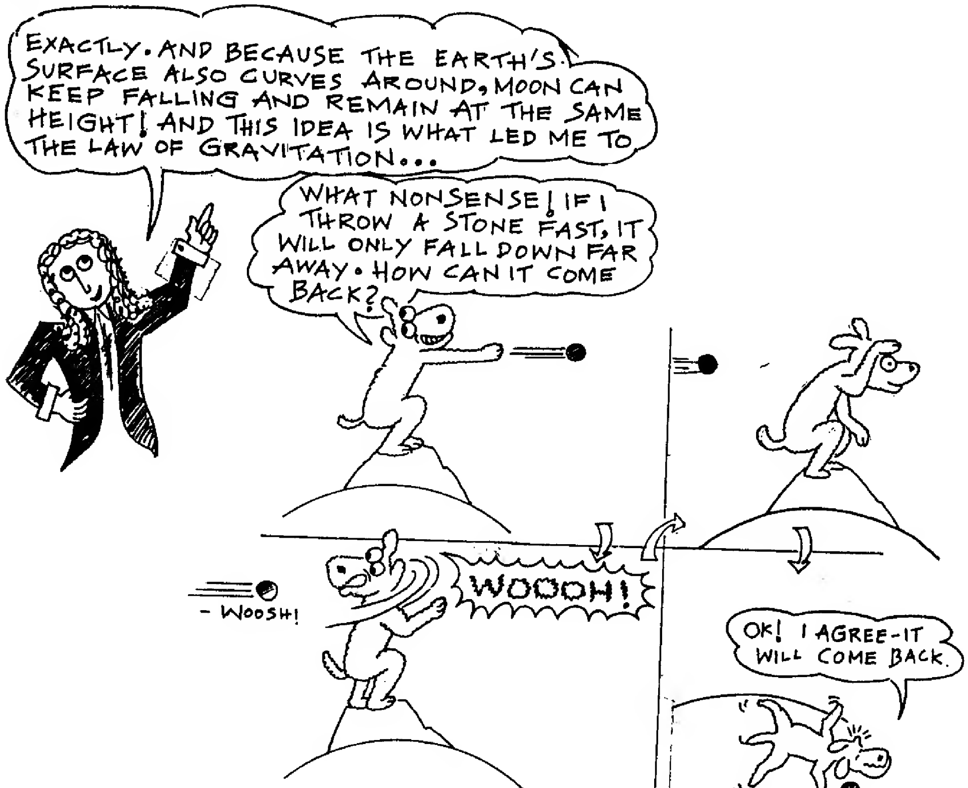


HERE AGAIN ALL THREE STONES FALL DOWN FROM THE STRAIGHT LINE BY THE SAME DISTANCE IN THE SAME TIME. BUT BECAUSE THE EARTH'S SURFACE CURVES DOWN, THE LAST ONE HASN'T STILL REACHED THE GROUND- IT IS STILL FAR ABOVE.

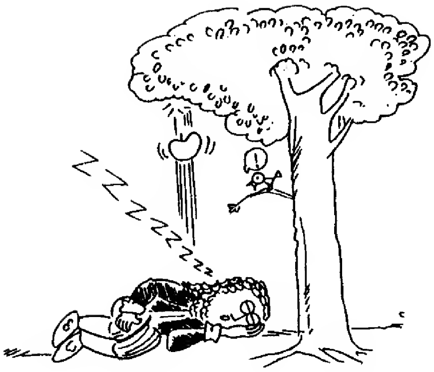
T: What happens if the stone falls and the earth also falls the same distance ? The stone has indeed fallen - but it is the same distance away from the earth's surface. Again it falls, and again the earth's surface falls away - this keeps happening till finally the stone is back to the place it started from! And there you have the stone falling all the time and therefore moving in a circle!



S: Ok - Now I get what you mean. The Moon is moving in a circle and so it is really falling towards the earth all the time. It is not falling in - as from rest. But it is falling in from a straight line.



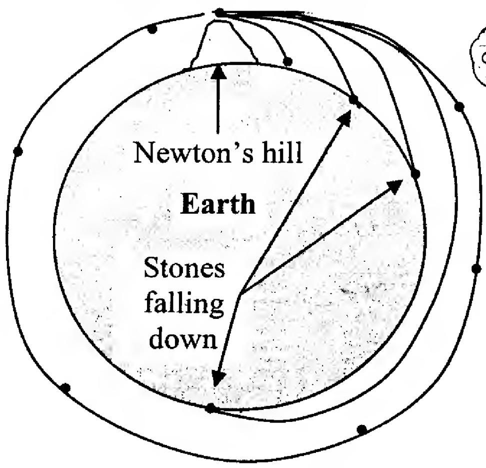
A bit of history...



When the apple fell on Newton's head his deep thoughts were not on why it fell, it was possibly on...

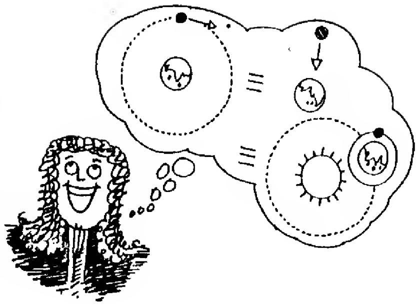
People keep talking of Newton and the Apple. That is nonsense. You think about why something happens only when it surprises you. Clearly Newton (or you and I for that matter) would expect an apple to fall down. Newton couldn't have seen an apple fall and started thinking of why it fell! Newton, Galileo and others knew that things fall down and possibly something is pulling it down. The real question was whether this was true beyond the earth - for example for the Moon, Planets and the Sun. Initially Newton too thought of the Moon as *moving* around the earth - as going around instead of being at rest. His emphasis was on moving and not on the around (which is the same as falling). But Galileo's observations (Newton's first law) on moving objects continuing to move in a straight line began to influence Newton's thinking.

He started seeing what we discussed in the beginning of this section. He stopped asking why the moon is *moving* around. He started asking why the moon is moving *around* - why the Moon is not going in a straight line. This logic led him to see Moon's circular motion as falling towards the Earth. In fact the earlier diagram - stones thrown from a mountain top and falling at different places and one going off in a circle is borrowed straight from Newton's Principia Mathematica (the famous book where Newton first described the basics of mechanics, gravitation and planetary motion).



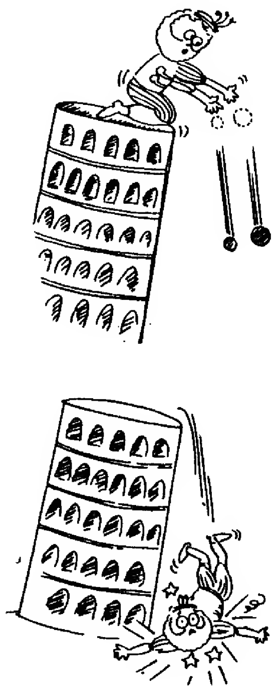
Circular motion is falling towards the Earth.

Once Newton saw that circular motion was the same as falling towards the Earth, he was able to see that a stone on the ground and the Moon so far away are both falling towards the Earth. Possibly the Earth must have something to do with it - maybe the Earth is pulling all objects towards itself. And that would not only solve these two problems! The Earth goes around the Sun in an ellipse. Our logic says that the Earth falls continuously towards the Sun and so do all the planets - so the Sun must be pulling all things towards itself. Generalizing from this idea Newton guessed (hypothesized) that every object must pull every other object towards itself.

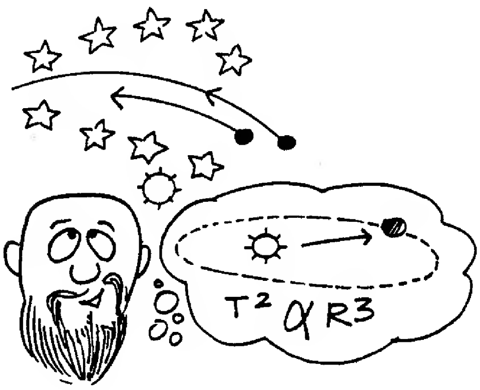


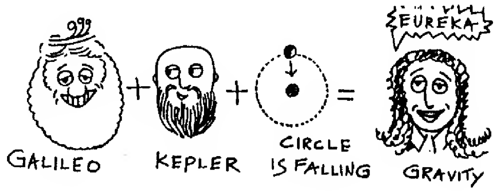
Galileo had shown earlier that the rate of falling on earth, of different objects was the same - the acceleration due to gravitational pull is the same for all objects. Newton's second law looks at what force does to an object and it says that force makes an object accelerate. The more massive the object, less is the acceleration. To keep the acceleration constant means the force must increase along with the mass of the object. This means the pull of the earth on the object must be proportional to the mass of the object. So reasoned Newton. Since Newton thought of this as a Universal force, it had to be true for any two objects and therefore the force should depend on both and not just one mass - so he guessed the force was proportional to the product of the masses.

Also it made sense that this pull (gravitational force) must decrease with distance. You affect far away things less. Before Newton, Kepler had discovered three laws of planetary motion. Newton did some simple mathematics and showed that if this attraction decreased as $1/r^2$, (ie. decreased as the square of the distance between the objects) then he could prove what Kepler had empirically shown for the planets.



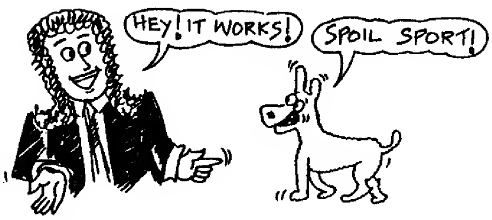
Galileo demonstrates from the leaning tower of Pisa, that all objects fall at the same rate.





Then Newton used this idea to test out whether the force really decreased as $1/r^2$. He knew the falling rate on the surface of the Earth $= g$ (about 10 m/s^2). He used the then current estimates for the Earth's Radius (R) and the distance to the Moon (d). He argued that the effective falling rate 'g_{moon}' at the place where the Moon is should be $g_{moon} = g\ (R/d)^2$. Moon takes a month to fall around and using this and some geometry one can show the falling rate of the Moon is $= d\ (2\pi/30\ \text{days})^2$. When Newton compared these two he found they did not match and a depressed Newton gave up his whole idea for a few years.

Then someone independently recalculated the distance to the Moon and found the old estimate was wrong. When Newton found this out, he quickly tried his calculation again - and this time it worked out perfectly. Back on track Newton worked on this idea and showed that with some of these basic ideas a lot of the world can be explained. This set the stage for the publication of Newton's three laws and the law of gravitation. Together these laws showed very powerfully for the first time that objects on Earth and in the heavens obey the same laws and that these simple laws can be used to explain a lot of the things we see around in the world.



Summary Points

1. Newton's hope that there are underlying patterns, that the Universe may have some laws governing its behaviour, is basic to all of science today. These laws are guesses - we hope they are true.

2. Focus on experiment - Newton guessed, but he also crosschecked his guess with the real world. And he was honest - if the facts did not fit in with his ideas, he was brave enough to reject the ideas. He did not alter the facts to suit his hypothesis. This is the most important cornerstone of the scientific method today. Experiment finally decides what is true.

3. Scientists like everyone else go through a simple thought process. It is not random inspired ideas that somehow strike scientists leading them to important results. It is instead a stream of thoughts, a building up of a mental model of the real world which finally helps them jump to conclusions.

4. Force is needed to not just speed up or slow down an object. Even to change direction of motion, we need force. Things moving in circles have a force directed inward - otherwise they would have gone in a straight line.

5. The first law - moving things keep moving in a straight line - required us to modify our view of what is natural for objects to do. Its power lies in the fact that with this simple modification we can now not only explain dusters moving on tables, we can also explain the motion of planets and satellites. This is something we see in all of science - small changes in ideas inside our heads (which sometimes maybe difficult to internalize) leads to a huge ability to explain the world around us. Science chooses ideas that make explanation simpler without compromising, manipulating or neglecting facts. This is what makes science powerful.

Comments, Questions and Experiments

1. Can a body remain at rest, if there is only one force acting on it?
2. How does a bird fly in air? If there was no air and the bird flapped its wings hard, what would happen? Why is it difficult for human beings to fly with attached wings?
3. A man climbs a tree. What gives the man the force to climb up? Why doesn't the tree move down?
4. If the two ends of a rope are pulled with forces of equal magnitude in opposite directions, why does the rope not move? Why is the tension in the rope not zero?
5. A horse is hitched to a wagon. Since the wagon pulls the horse back as much as the horse pulls the wagon forward, why doesn't the wagon remain at rest however hard the horse pulls?
6. After reading Chapter 1, a student comments "So when I walk, I push the earth, the earth pushes me back and so I move. But since I push the earth equally backward, why doesn't the earth move?" The earth does indeed move but by a very small amount. Let's say the student weighs 100 kg! And walks all around the earth twice! Using the idea that the student's mass times the distance the student moves must equal the earth's mass times the distance the earth has moved, show that the earth moves around is of the order of a proton's diameter (10^{-15} m). Using this show that even if all the human beings on earth (6 billion people) were to stand in one line end to end and walk in the same direction walking around the earth twice, the earth would only move by one-hundredth of a millimeter! The mass of the earth is 6×10^{24} kg and the radius of the earth is 6400 km.
7. How can pushing the cycle pedal down, make the cycle move forward?
8. A heavy weight can be slowly pulled up by a thin string, but when the string is jerked it breaks. Why?
9. Can a woman jump up on the moon? Can she do this in deep space?
10. Imagine two people pulling and pushing in space. What will happen?
11. Unlike airplanes, rockets can move in deep space. How do they do this? If a rocket in space wants to change its direction, how will it do this?
12. An astronaut in a space station goes out for a 'space walk' - but has forgotten to tie herself with a rope to the station. How can she come back?
13. Can you stand without moving on a slippery oily floor? Can you walk on an oily floor? Why do people who try to walk slip on an oily floor?
14. When you place a book on a table, it does not fall down. But the earth is pulling it. Why doesn't it fall down? You might have read in books about Normal Reaction from the table on the book. Is this Normal reaction really a reaction? If so what is it a

reaction to? The book's weight is really the earth's pull (force) on the book. What is the reaction to this force? If the 'Normal Reaction' by the table is not the reaction to the earth's pull on the book (and it is not - so if you said it is the reaction, change it!), then what is the reaction to this 'Normal Reaction'? To avoid this common confusion, it is much better to call this Normal Force.

15. If the sun pulls the earth, the earth also pulls the sun. So why doesn't the sun go around the earth? (Think about this. The sun actually also goes around. Both the sun and the earth revolve around a point on the line joining the two. But this point is very close to the sun's center and in fact within the body of the sun because the sun is so much larger.)

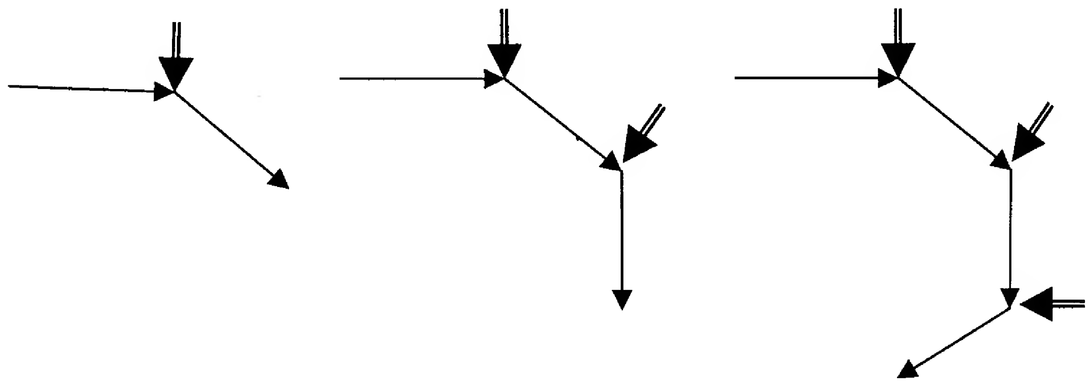
16. When a car stops suddenly, the passengers are thrown forward, away from their seats. Why?

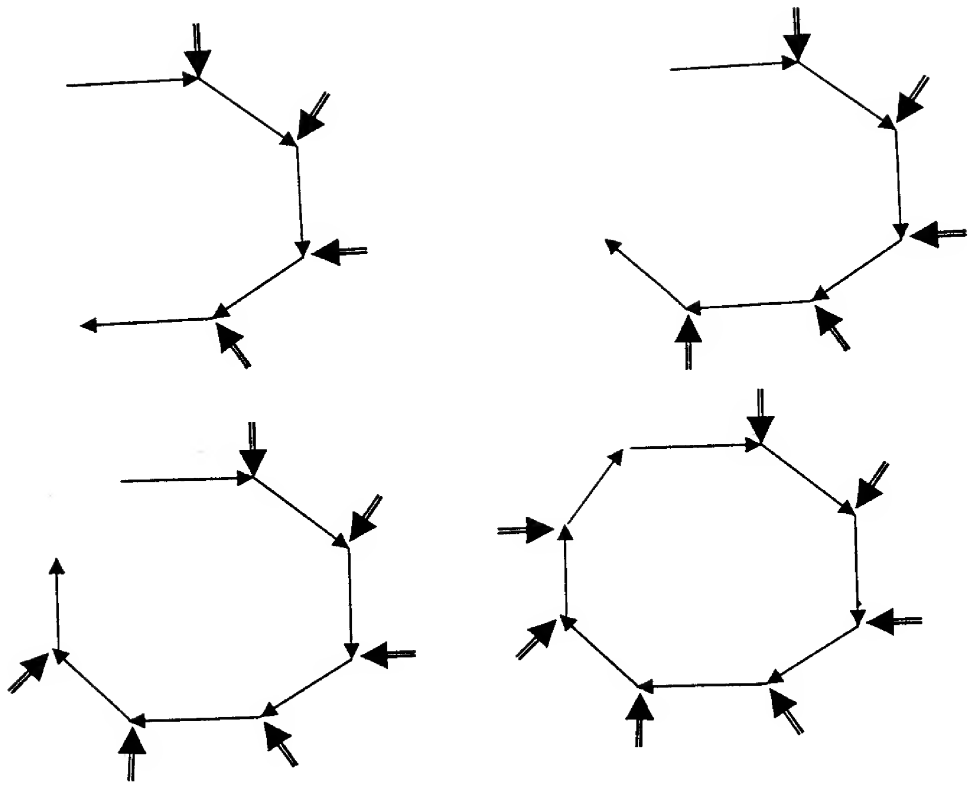
17. When a bullet is fired from a gun, what is the origin of the force on the bullet?

18. Since the earth is constantly attracted by the Sun, why does it not fall in and burn up?

19. A student claims that if a 100 gm apple falls down from a tree 3 m high, the apple must be attracting the earth and the earth should also fall up and they should meet midway. The mass of the earth is 6×10^{24} kg, the radius of the earth is 6400 km and acceleration due to gravity on the surface is $10\ m/s^2$. Using all this, find out how far the earth moves up when the apple falls down.

20. We saw how circular motion is like falling in. Newton showed this very beautifully. Consider an object moving in a straight line and in brief intervals I give this object an impulsive push perpendicular to it as shown in the figures below. Because of these forces the object moves in the path shown. In between these impulses, the object continues moving in a straight line with constant velocity according to Newton's first law.





The above seven figures show how in each step the direction of motion of the object changes and the object comes back to the starting point. If we reduce the time between the impulses, the number of sides in the polygon figure will increase and it will look more and more like a circle. Each of the impulses is inward and when the time gap between impulses becomes very small (zero), all the forces will continuously point towards the center. So a continuous pull from the center will make the object go in a circle and the object is really *falling inwards* all the time it is moving in a circle.

21. There is an interesting experiment that can be tried out at home or in class. Cut a small piece of a straw (a fat straw is better) and pass a thin string through it. Then tie the string across the room. The straw piece should be able to slide on the string without much difficulty (friction). Now blow up a balloon and with a cellophane tape tie it to the straw. Now release the mouth of the balloon and the balloon should rush from one end of the string to the other. Why? This is the rocket principle.

22. This is another experiment that you can try. Place a bunch of carom coins one on top of the other. Now carefully strike the bottom most carom coin with another carom coin. The new coin will replace the earlier one and the tower of coins does not fall. You will have to do this with some care and practice. You can also do this with a bunch of coins. You can instead place all the coins on a piece of paper and pull out the paper very fast without disturbing the tower. If you pull the paper slowly the tower moves with the paper, or shakes and falls. Do this experiment and then think about why it happens.

Chapter 3

Newton's Second Law

In the first two chapters we had a dialogue on Newton's third and first laws. We looked at how these laws must be understood and the rationale behind these laws. This chapter is more in the form of a lecture with problem-solving techniques rather than a dialogue. We first take a brief look at Newton's second law. Then we see how Newton's laws can be applied to specific problems. This chapter assumes that you have some basic knowledge of algebra and vectors and is mathematical. In the last section, once you are comfortable with using Newton's laws, we look at the meaning of these laws.

What Newton's Second Law says...

The rate of change of momentum of a body is proportional to the net Force on the body.

Momentum of the body = mass × velocity of the body = **mv**. And since normally the mass does not change, momentum change means velocity change. So rate of the change of momentum means mass times rate of change of velocity. Rate of change of velocity is acceleration and so Newton's second law says that **F α ma**. This means **F = kma,** where k is a constant. We choose the unit for Force such that k is 1 and so we get the famous equation **F = ma.**

So far this is just a description of what Newton's law says. But the real question is whether this Newton's second law[1] makes any sense. Let's discuss this.

T: What is force?

S: Force is a push or a pull.

T: So what does push or pull do?

---

[1] *Students who do problems in pulleys, inclined planes and dynamics often see Newton's second law as the most difficult of the three laws. Actually this is not so. It is, in many ways, relatively the easiest. The first law is the hardest to grasp - it is also the hardest to demonstrate and fully understand. The third law is again easily misunderstood. The problems with the second law are usually problems with the other two laws that carry over to this law. The tricky part of Newton's second law is in finding the net force - adding a whole lot of forces. This adding is difficult because you have to add arrows (vectors) - not just numbers - because force comes with a direction and can cancel or partly cancel. But all said, writing down Newton's Second law for any object is reasonably easy. Solving it is another matter though! To fully understand the second law, one must understand the first and third laws. As mentioned earlier the three laws are one package. Together they explain the world - individually they make no sense.*

S: It changes how the object is moving.

T: But this is vague. What do you mean by "changing the object's motion?"

S: For example, making an object at rest move.

T: Watch out! You need force initially to get it moving, but to keep it moving you don't need any force. Newton's first law says that objects naturally keep on moving forever uniformly in a straight line. The real meaning of 'changing the object's motion' is a change from its 'natural motion'. Any change from uniform motion in a straight line is caused by force. But uniform motion means constant unchanging velocity. So what changes when you change from 'natural motion'?

S: The velocity changes.

T: Precisely. This means a push or pull changes the natural motion (which is unchanging velocity). So force changes velocity. The next question is how does it change the velocity?

S: By increasing it.

T: Well, it could decrease the velocity as well. Force can stop objects. But that's not what I am asking. Does a force add a fixed amount to the velocity or what does it do?

S: Force causes acceleration - the time rate change of velocity.

T: Let's see if that makes sense. If I push for longer, the object's velocity changes by a larger amount, doesn't it? This means it is not just the amount of force, but also the amount of time that matters. So more time and more force means more change. So it seems reasonable to say that change in velocity depends on Force $\times$ time. This is the same as saying that "the change in velocity per unit time (acceleration) is the real effect of the force". So acceleration is proportional to the force ($\mathbf{a} \propto \mathbf{F}$).

S: So why then is $\mathbf{F} \neq \mathbf{a}$, why is it $\mathbf{F} = m\mathbf{a}$.

T: Because **a** is not just proportional to **F**. With the same force if you have a larger mass, the acceleration is smaller. If you try to push two objects with the same force (that is equivalently one object with twice the mass), you get half the acceleration. So acceleration is also inversely proportional to the mass - the more the mass correspondingly lesser is the acceleration. $\mathbf{a} \propto 1/m$. Together with $\mathbf{a} \propto \mathbf{F}$, this becomes $\mathbf{a} \propto \mathbf{F}/m$. This is the same as saying $\mathbf{F} \propto m\mathbf{a}$. Then we choose the units so that $\mathbf{F} = m\mathbf{a}$.

S: But just like $\mathbf{a} \propto \mathbf{F}$ and $\mathbf{a} \propto 1/m$, maybe **a** is also proportional to other things. In that case won't they also come into the equation? How do we know it doesn't happen?

T: You are right. If you could show that **a** also depended on other things, then we have to change this equation. Physics and Science cannot be done entirely through arguments! Logic **plus** experiments decide what laws work in science. Logic alone would leave us with many alternatives. We choose a reasonably simple one and test it with a whole lot of experiments. If the experiments agree with the guess it becomes a law. In this case, we know anything less than $\mathbf{F} \propto m\mathbf{a}$ won't work. Even simple arguments so far have shown this. But as you say

maybe we need more. We start with this and see if this works. And surprise - it works wonderfully well! So this then is the law we use.

Hopefully it is now clear why **F= ma**. In the next section we will apply this law to specific problems and show how to use this law and the other two laws of Newton.

The Grand Plan

Newton's laws by themselves are of little use. Along with these three laws, we need force laws - statements that tell us what the force in different situations is. Force laws for different phenomenon like gravitation, electricity, magnetism, etc tell us what the force between objects is. We use these laws and Newton's second law to find out the acceleration of the body. Why do we need acceleration?

The starting point for all physics is motion. If we knew how every little thing in the universe moves, then we know where things are, what they will do next and so we can predict the future. Once we know this we are done. We don't need forces. We don't need accelerations. We don't need to have separate theories for gravity, electricity, etc. Of course it is not easy to predict the motion of all things. That's why we have all these diverse subjects - like light, electromagnetism, heat, etc. But at least in principle, the knowledge of the motion of all objects (including atoms inside the objects and electrons inside the atoms) is the ultimate sum total of all our physical knowledge.

S: How can you say that? Are you saying everything in the world is just motion?

T: Yes - in a sense. Usually if you burn a piece of paper or if you cook something, you wouldn't think of it as just motion. But these are merely chemical reactions. But at the atomic level, it is just the motion of atoms and electrons. Earlier the electron was moving around one atom and now it is moving around two atoms. This means the two atoms are one object now - they have formed a molecule and this leads to its different properties. Similarly we talk about solids, liquids and gases. This classification seems to have no connection with motion. But it does. If you look at the motion of the atoms and see them merely vibrating about fixed positions, the object is a solid, if the atoms slip away and move around, it is a liquid and if the atoms fly away from each other almost independently of each other, it is a gas. Just by looking at the motion, you can say what kind of object it is - in fact molecular motion pattern is what determines the state of the object as a whole. Coming to human endeavors, let's say you walk into a house, meet your girl friend, chat for one hour, and then walk out, meet another friend and go to a movie. If we plot in detail your motion, your girl friend's motion and the other friend's motion, you know the sequence of what happens. If I want to know exactly what you and your girl friend talked, all I need is the detailed motion of your tongue! Motion is everything!

T: What form will this description of motion take? How will we describe how things move?

S: By saying where each object is.

T: Yes, but it is not enough to just say where each object is **now**. You need to say where each

object is **every** instant of time. Complete description of motion means specifying where the object is at every instant of time.

S: But that's a very hard thing to do. There are so many instants - how can we write down where the object is at every instant. We cannot write this down even for one object.

T: That's why we have a mathematical tool for talking about such descriptions. They are called functions. Functions are formulas - you tell me the time and the formula will tell you where the object is at that instant of time. Instead of writing down infinite time instants and for each instant writing down the position, you write one formula - with which you can calculate the position at any instant.

S: You mean for each object there is formula for where it is?

T: Yes. Usually we plot this on a graph - position versus time. For each formula you get a specific curve. This curve is the graphical way of talking about the formula. They are both the same thing. The graph is not the way the object moves around in space - it is the way the object moves in time. This graph is usually called an x versus t graph (or a y versus t or **r** versus t graph).

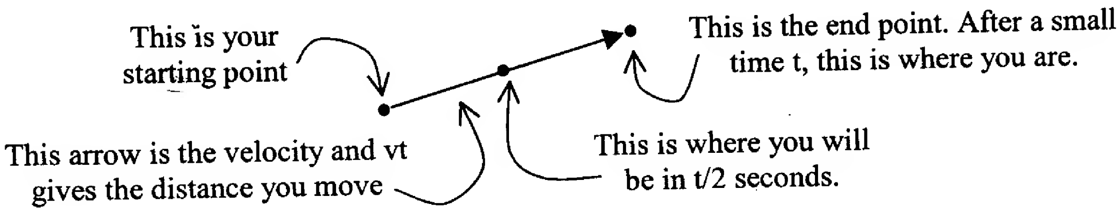
Time	*Position*
0.0 s	0.0 m
0.5 s	0.25 m
1.0 s	1.0 m
1.5 s	2.25 m
2.0 s	4.0 m

$$\Longleftrightarrow \quad x(t) = t^2 \quad \Longleftrightarrow$$

Table of position at every instant of time, formula for x as a function of t and a graph of x versus t are just different ways of describing the same motion

S: So the purpose of physics is finding these x versus t graphs or formulas (functions)?

T: Yes. But finding these x(t) functions is a hard job. Luckily, if you know the starting position and the velocity at that instant, you can find the position at the next instant. Your velocity v tells you how much you move in a small time t (*the distance traveled is vt and is in the direction of v*). You know where you started and you know how much you have moved. So you know where you are now!



S: Oh! So you mean from the new point, I can again calculate the next point from the new position and velocity. That means with just the initial position and velocity, I can find out the entire motion, just by calculations!

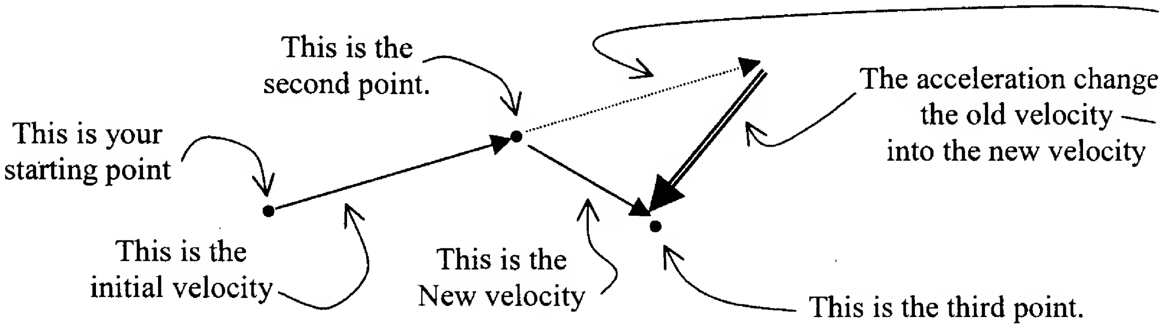
T: Ah! If only that were true! But unfortunately, the velocity at each point also changes. If the velocity were the same, you can indeed calculate where you will be at every instant. But since it changes, you need to know the velocity at the new point to find the next point, and the velocity at the next point to find the point after that, and so on.

S: But this means I need the velocity at every instant of time?

T: Exactly.

S: But that means I need a formula for the velocity. In that case, why bother about the velocity in the first place. I might as well find the formula for the position.

T: In a sense you are right. But have some patience. Maybe we can calculate the velocity at the new point. That's what acceleration does. The acceleration now tells you how the velocity changes and so it tells you the velocity at the new point. With this you can find the velocity at the new point and therefore the position at the third instant.



S: But this again will work only if the acceleration is constant. What if the acceleration changes at every point? Won't it mean we now need a formula for finding the acceleration at every point?

T: Yes - it will mean we need a formula for finding the acceleration at every instant of time. But surprise! We already have this formula: **F = ma !**

S: Oh! So you mean, we use **F = ma** to find the acceleration at each point, and then use the initial velocity and this acceleration to calculate the velocity at each instant and then use the velocity at each instant and the initial position to calculate the position at every instant of time.

T: Exactly. That's the grand plan and that's why finding the acceleration is so important.

S: But isn't this a lot of work and calculations?

T: It is. But with computers, we can do such calculations very fast - that's how rockets are launched. There are lots of calculations - but these are just routine calculations and computers can do them easily. Before computers, people did this manually. But they also invented

calculus to do these kinds of calculations - it helps solve at least a few problems without all this calculations. But for really complex calculations, one must go back to the basic step by step computing method.

S: Why is finding acceleration so much simpler than finding velocity or position? Life would have been lot more easier if we could find the position directly or at least the velocity.

T: Good question - but I don't know the answer! It just happens that when you search for forces in terms of accelerations - they look simple. Nature just doesn't seem to have good simple formulas for positions and velocities. She seems to have good formulas only for accelerations. We just have to accept it and work with it.

Anyway, now that you know why accelerations are so important, let's figure out how to calculate and find the accelerations in different problems.

Applying Newton's Laws...

These three laws (along with some more force laws) form a very powerful package and almost the whole world we see around us can be explained to fair degree with these laws. But doing that is going to be hard and will take a long time. So instead in this book we will focus on a few simple examples to show you how to 'solve' simple and artificially created problems.

S: Why artificial problems?

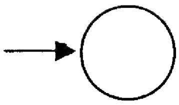
T: In principle Newton's laws can and does help solve real problems. But real problems are very complex - even simple ones like a glass of water slipping and falling down. There are just too many things to worry about - the shaking of my hand, the wind, the angle at which the glass is tilted, the amount of water, etc. But with all this complexity and our current level of mathematical sophistication, we will only be able to talk in general terms without being able to quantify. Instead, what we will do here is to simplify the problem and quantify our answers.

Free Body Diagrams

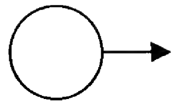
Before we get to solving problems, I will set out our framework. This framework is called the free-body diagram framework. This is what it is...

1. We have several objects interacting in each problem. The first job is separating each object and making it "free". This means you replace all the interactions on this object by the other objects, by forces. So the diagram looks like a free object with a lot of arrows on it. You should do this for every object in the problem. On the free body, the only arrows should be forces **on** the body.

   Convention: Draw all forces arrows away from the body. So even if the force is pushing, you show it as an equivalent pull starting from the object. In simple one dimension problems, this is not necessary. But in more complicated 2-dimensional problems with several forces, following this convention makes things easier.



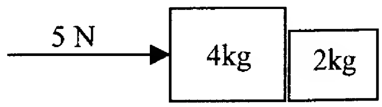
You replace the force as shown on the left by the arrow on the right.



2. In replacing other objects by forces, you should use Newton's third law.
3. Once you have used Newton's third law and drawn the free body diagrams for *all* the objects, add up all the forces on each objects (vectorially) and then use Newton's second law to find the acceleration of the object.
4. There are three critical steps in solving any problem...
   a. **Kinematics** - Figuring out how the object moves and any relationship between the motion of various objects. There is no systematic procedure for doing this. This is where you need to develop a feel for the problem and imagine what happens.
   b. **Dynamics** - Free body diagrams and Writing Newton's Laws - this is very simple and straightforward. Just separate the objects, add the forces on each body and equate it to **ma**. This gives you a set of equations.
   c. **Mathematics** - Solve the equations that you get from Newton's laws.

The best way to understand this framework is by actually solving a number of problems. So that's what we do now...

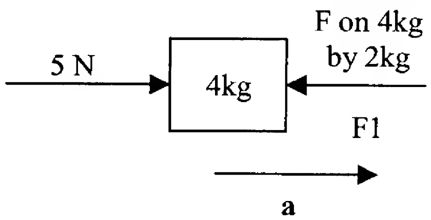
Problem 1: A space-woman pushes a 4kg block which is next to a 2 kg block as shown in the figure with a 5 N force. What is the force on the 2kg block ?

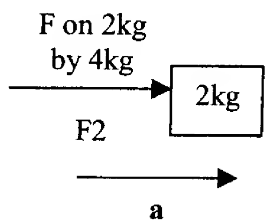


Solution:

Kinematics: How will the two bodies moves? Obviously from experience you know they must move together. The force pushes the 4 kg, which pushes the 2 kg block and the 2kg moves along with the 4kg block. That means their accelerations must be the same. (Why? Think about it). Call this common acceleration **a**.

Dynamics (Free body diagrams and Newton's Laws): The first step here is separating the objects and replacing interaction by arrows (forces). The space-woman is already shown by an arrow. So let's start by separating the two blocks. On the 4 kg block, there is one force by the space woman (5 N). But if you want to get rid of the 2kg block, then you must replace it by another arrow. So that is what we do below. Similarly the 2kg block is interacting with the 4kg block, so if you want a free body diagram for the 2kg block, you must remove the 4kg block and replace it with another arrow.





Now by Newton's third law we know that Force on 4kg by 2kg block is equal and opposite to the force on the 2kg block by the 4kg block. But if you see the diagram we have drawn the arrows already opposite, so the magnitudes must be equal => F1 = F2 (call this = F). With experience, you should start doing this as you draw the free body diagram itself, so that you can save time and space.

Note that we now have each body separated out with only a bunch of arrows on it. This is what we call a "FREE" body diagram. And see how I have marked the acceleration. It is close to the object - but not touching the object. The only arrows touching the object are the forces **on** the object. This is very important. Don't mark accelerations with arrows on the objects. Draw it separately (near the object, but not on it). So when you want the net force, it is just the vector sum of all the forces touching the object. If you draw the acceleration also touching the body, there is a temptation to see **ma** as a force and to add it with the rest - and this is wrong.

Also on the free body you are worried only about the forces on the body - once you mark the force you don't have to worry about what caused that force. The force has all the information- the cause does not matter. (Remember the zeroth and halfth laws!)

Writing the basic equations:

Now we have separated the two interacting objects. We only have forces on them. So let's apply Newton's Second Law to the 4 kg block.

Net force on the 4 kg block = 5 - F (towards the right) = m × a = 4 × acceleration of 4 kg block.

Applying the second law to the 2kg block we get

Net force on the 2 kg block = F (towards the right) = m × a = 2 × acceleration of 2 kg block.

But our earlier kinematics part told us that both the accelerations are the same = **a**. So we get the following two Newton's equations:

$$5 - F = 4a \quad \text{and} \quad F = 2a$$

We have two equations and 2 variables. We can solve this easily. So we move on to the mathematical part of the solution.

Note that I wrote Net force on the body = **ma**. Sometimes students write **F - ma = 0**. This should be avoided when you are writing the basic equation. **F = ma** means **F causes ma**. It is a physical statement of cause and effect. When you write **F - ma**, you are treating **ma** as a force - which it is not! This is not a mere pedantic point. Many students draw acceleration as an arrow on the block and count it as a force. Because they see **ma** as a force, they usually add it up along with other forces and actually get **F1 + F2 - F3 - F4 + ma = 0**, when it should actually be **F1 + F2 - F3 - F4 = ma**. So the equation itself (and therefore the solution) becomes wrong. Also this leads to a lot of unnecessary confusion and loss of confidence. Newton's law are very clear and simple. Find the total **F** on the object (add vectorially all the forces). This causes **ma**. So it is important to write the equations like this. There is less chance of making mistakes.

Mathematics: In this case the mathematics is very simple. Just two simple simultaneous equations. Solving it, we get **a = 5/6 m/s²** **and F = 5/3 N**. So the force on the 2 kg block is 5/3 N.

The above was a relatively simple problem. We will slowly move on to more complex problems. But the key is to recognize that there are three steps in solving any problem - Kinematics, Dynamics and Mathematics. The first is intuitive and creative, the second is a formal procedure and very straight-forward and the last is mathematical jugglery and manipulation. Mastering this technique gives you a powerful way of thinking (and calculating) about the world.

Before we continue with more problems, let's look at some basic objects that we will come across and the forces associated with them. For now we will restrict ourselves to nice frictionless surfaces and objects. So there are blocks, surfaces, ropes and pulleys. Let's see what these various objects do...

Ropes and Tension

First let's look at a rope. Let's tie it to a wall and pull it.

Now, you can't push with a rope. But you can pull with it. This is the first lesson in working with ropes. Actually with a real thick rope, you can even push a bit. But it is hard. So to make things simple, we create and deal with ideal ropes! These ropes can only pull, never push. For now, we will also assume they are massless ropes and smooth frictionless ones which don't break easily or stretch very much. When you pull a rope that is tied to the wall, what happens? You feel the rope is "tense" or "taut". We say the rope is in tension. What is tension?

T: The simplest answer is this "Tension is the force with which the rope pulls."

S: Pulls what ?

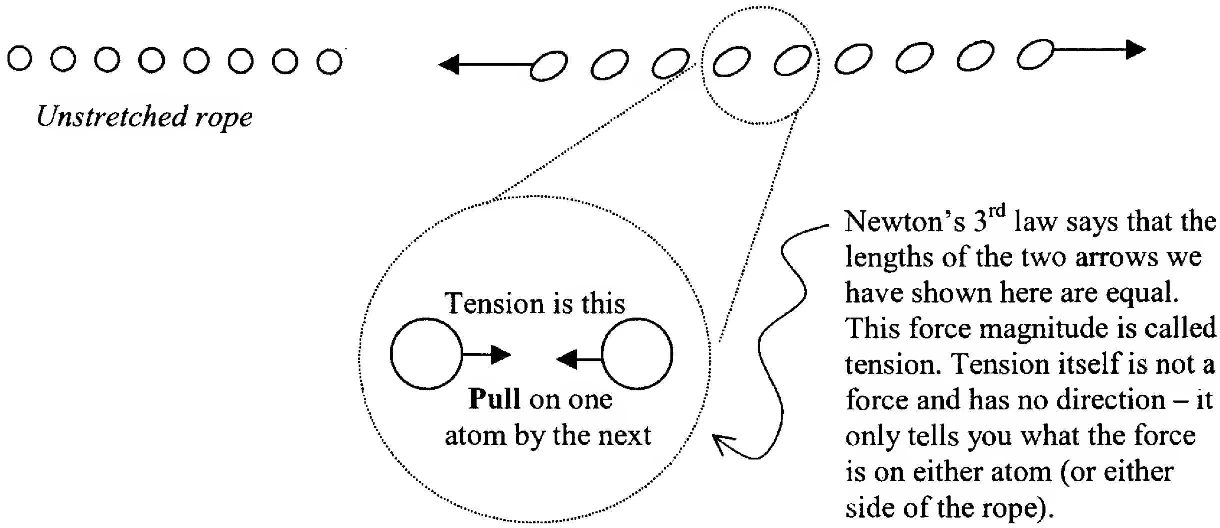
T: Good question. Since I am pulling the rope, the rope must be pulling me - so that is the rope's tension at my hand.

S: But the rope is also pulling the wall. Is that also the tension?

T: Yes. That is the rope's tension at the wall.

S: You mean there is different tension at different places?

T: Can be different if needed. In this case, the tension is the same. But yes - often the rope's tension is different at different points on the rope. Let's try and understand what happens inside the rope. I will draw a picture of a simple ideal rope just to help you get a feel. A rope is made of a row of atoms. These atoms pull each other, but if they get too close, they push. This way they try to keep themselves at a fixed distance. When you pull the rope, the distance between atoms increases slightly, so they pull each other. This force with which they pull each other is the tension. (Actually it is not the force between one atom and another - but rather between all atoms in one cross-section and the next cross-section).



S: How much is this tension?

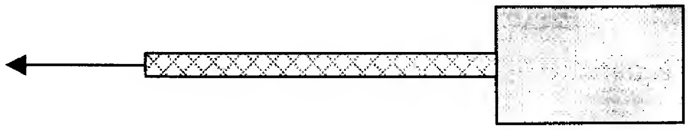
T: Let's use Newton's laws to find out. Let's consider the part of the rope that is to the left of the atom shown above. On this object (rope) there are two forces - the tension force by the other side of the rope and my pull on the rope.

T: If the rope is massless (or if it is not accelerating), then **ma** of the rope = 0. This means the net force on the rope must be zero, therefore the Tension **T** = **F**. And since we chose a random point, this is true for all points on the rope. So the rope pulls the wall with the same force as my pull on the rope.

S: But this means tension is the same only for a massless rope. What happens if the rope has mass?

T: It is true if **ma = 0**, so either m or a has to be zero. If it is a rope with mass which is accelerating, then the tension is the rope is not the same everywhere. Newton's third law says that the tension immediately where I am pulling is equal to my pull. But as you move down the rope, this will no longer be true.

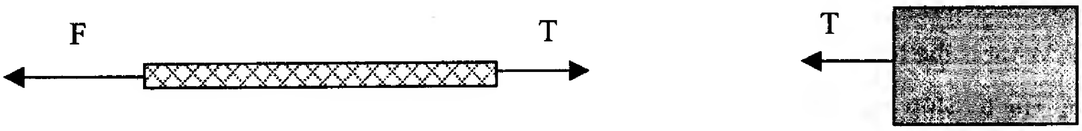
Problem 2: As shown in the figure, our space-woman again exerts a force **F** on a block of mass **M**, but this time she pulls it through a rope. What is the tension in the rope mid-way, if the rope is massless and when the rope has a mass equal to **m?**



Solution:

Kinematics: As before, both the rope and the mass move together - so their accelerations must be same whether the rope has mass or does not have mass. Call this acceleration **a**.

Dynamics: As always the first job here is separating the objects and drawing the free body diagrams. We do that as shown. And even as we draw the diagram, we have used Newton's third law. (Can you see where?)



Part A: When the rope is massless.

Writing the equations: Using Newton's second law for the rope and the block, we get

$$\mathbf{F - T = m_{rope}\, a = 0} \qquad \text{and} \qquad \mathbf{T = m_{block} a = ma}$$

Mathematics: Solving the two equations we get, **T = F** and **a = F/m.**

F is the tension at both ends. One can break up the rope midway and find the tension there and it will also turn out to be **F**. Or one can argue that since the tension is the same throughout (why?) - it must be **F** at the midpoint as well.

Part B: When the rope has a mass m

Writing the equations: Using Newton's second law for the rope and the block, we get

$$\mathbf{F - T = m\, a} \qquad \text{and} \qquad \mathbf{T = M\, a}$$

(In writing this, we used the knowledge from the Kinematics part that **a** is the same.)

Mathematics: We have two unknowns (**a** and **T**) and we have two equations. Solving these we get

$$\mathbf{a} = \mathbf{F/(m+M)} \qquad \text{and} \qquad \mathbf{T} = \mathbf{FM/(m+M)}$$

But we need the tension midway, so let's mentally divide the rope into two halves and draw the free body diagram for one half...

Dynamics:

Writing the equations: Applying Newton's 2nd law to the left half of the rope, we have

$$F - T_{mid} = (m/2)\,a.$$

Mathematics: But we know **a**, so putting that in and solving for $\mathbf{T_{mid}}$, we get $\mathbf{T_{mid} = F\,(M+ m/2) / (m+M)}$. So Tension everywhere on the rope is not the same.

In these simple problems breaking up the solutions into three parts looks silly. But as we get to the more difficult problems, this is the procedure that will really simplify matters. So it is good to learn it systematically at this stage itself.

T: What is the tension in a massless rope if I tie the rope to a wall and pull it with a force **F**?

S: The tension is **F**.

T: Right. Now what is the tension in the rope if I pull the rope with the same force **F**, but instead of tying it to the wall, you pull it on the other side with a force **F**?

S: Tension is **2F**.

T: That's what many students feel - and it is wrong! The tension is still **F**. Tension is the increased pull between atoms - because you have stretched the rope. But how can you pull the rope only on one side? Whether you physically pull on both sides of the rope, or tie one side of the rope to a wall - there is a force **F** on both sides of the rope. We generally feel that we are pulling, and the wall is not. This is not true. The wall pulls as well as we do - only it does *hee* and *haw* like us and so we don't see it as a pull. So the tension is **F** only because both sides pull with **F**! Think carefully about this - and about what tension really means.

Now let's look at another kind of force - the force between surfaces.

Surfaces and Normal Forces

T: One of the first things we notice even as kids is that the earth pulls us down *with a force.* We call this weight. If an object's mass is **m**, its weight is **mg** (where **g** is the acceleration due to gravity). If the earth pulls you down why don't you just fall through the floor ?

S: Because the floor stops me from falling. It pushes me up.

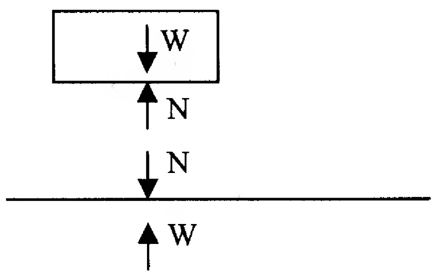
T: Exactly. But why does the floor push you up? You are made up of a bunch of atoms and so is the floor. Because the earth pulls you, your atoms try to go down. When they do this they come very close to the atoms of the floor. Now atoms are made up of positively charged protons and negatively charged electrons. When two atoms are quite far apart, you can think of atoms as being overall electrically neutral. There is no force between them because the force by the proton is cancelled by an opposite force by the electron. But as two atoms come very close, the forces between the electrons and protons don't balance anymore. Therefore when your atoms get too close to the floor's atoms there is an effective electrical force on your atoms pushing them up. Atoms generally like getting close to each other, but not too close - when you get too close they push you away. This push is an electrical force, though it doesn't seem like it.

S: You mean the floor's atoms push my atoms through electric forces?

T: Yes.

S: But I thought the floor exerts a Normal Reaction on an object.

T: This electrical force ***is*** the Normal Reaction. It is not a reaction. It is a force. Unfortunately someone called it normal reaction and the name stuck. Many students think of Normal Reaction as the reaction to the weight of the object. This is wrong. The reaction to my weight (the force with which the earth pulls me) is my pull on the earth. Normal force on the other hand is another force **on** me.



In the figure above there are two sets of action-reaction pairs. Weight and the body's pull on the earth is one set and the two normal forces is the second set. The **W** forces are gravitational in origin and the **N** forces are electrical. If they happen to be equal, the body does not accelerate.

T: When you are on the floor and not accelerating up or down, **W = N**. This does not make **W** and **N** an action-reaction pair. If you are falling from the top of a tree, there still a **W** on you, but there is no **N**. This **W** makes you accelerate downward. Where is the reaction to this **W**?

S: The **W** on the earth.

T: Exactly. The two **W**'s are an action-reaction pair. They always exist together. **N** and **W** are not action-reaction. If they happen to be equal, the block won't accelerate - that's all.

S: If the **W** on me makes me fall, won't the **W** on the earth make it go up?

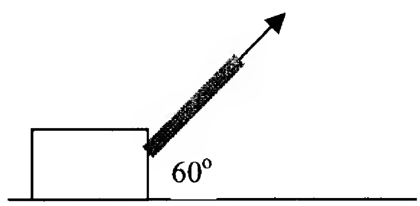
T: Yes. Every time you fall, the earth comes up to meet you. But the mass of the earth is so much more that it will move very little (much less than a proton's distance). You can never measure or feel it, even if all the human beings on the earth jointly fall from a building top.

T: Coming back to the Normal force... In most cases, the floor's force is not Normal (ie. not perpendicular) to the surface. It is at an angle to the surface and depends on how the body tries to enter the floor. Remember, the floor's force basically arises because it wants to stop the body from passing through the floor. So depending on how we push the object the floor pushes back. But the floor can't push at all angles - it usually can push within a small cone close to the Normal. We usually break up this single force from the floor (or a surface) into two parts - Normal force and a tangential force which we call friction. In this book, we will leave out friction and only consider frictionless surfaces. That means the force by the surface or on the surface has to be normal.

This force between block and surface is something that we come across whenever two objects are pressed against each other. One object doesn't have to be the floor! When two objects collide or when you throw an object and it hits a friend - in every single case the force between the objects is electrical and is like this Normal force we have just seen.

So far, we did not really use vectors. Now we start using vectors in a big way. There is a note on vectors at the end of this chapter. Take a cursory look at this problem, then read the note and come back to this problem.

Problem 3: The block shown in the figure is pulled with a rope at an angle of 60° to the horizontal. The rope is massless and there is a force of **F = 10 N** pulling it. The mass is **m = 2 kg** and the floor is frictionless. What is the Normal force by the floor on the block and what is the block's acceleration? (**g = 10m/s²**). What is the Normal force and the block's acceleration when **F = 30 N**?

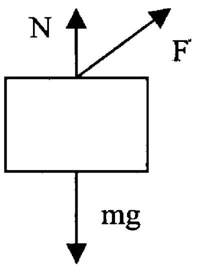


Solution:

Kinematics: How will the block move? Will it move along the rope? Intuitive feel for the problem says that the rope must move horizontally. We have often dragged toy cars or carts with ropes like this in childhood! If our pull is not very strong, the toy moves on the ground.

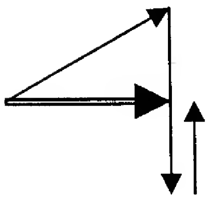
But if we pull very hard the toy moves up as well. Our Kinematics intuition says there are two possibilities. Either the block moves horizontally or it moves upwards. If it moves horizontally, the block is touching the surface and pushing it and there will be a normal force on the block by the surface. But if the pull is very strong the block starts moving up, then it won't push the surface down and so there is no normal force on the block. It is important to do this simple analysis before we start writing Newton's laws and solving equations. Let's assume for now that the pull of **10 N** is not large enough to make it move up - that means it moves only horizontally. If things don't work out, we can change the assumption.

Dynamics & Mathematics: As always the first step is drawing a free body diagram. Since the rope is massless, the tension is the same throughout the rope and is equal to the force pulling it at the top end – **F = 30N**. So we can just draw the free body diagram for the block.



On the block there are 3 forces. The earth's pull (weight) = mg pulling the block down. The surface's Normal force upwards=N and the rope pulling the block with F at an angle of 60°.

We have to add these three forces vectorially to get the net force on the block. Now, we know the direction of the net force is the direction of the acceleration and that is horizontal. So the sum of the three arrows must be horizontal.



In this figure, the three forces are added together tail to head. The double arrow is the resultant (net) force. And we know it has to be horizontal – because the acceleration is horizontal.

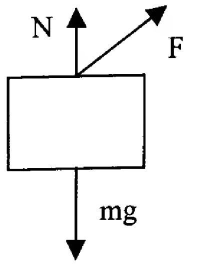
The upward contribution of F (= F sin 60°) makes the arrow go up, mg makes it go down and N must be such that it again brings it back to the starting height. Therefore mg - F sin 60° = N

The total horizontal force (in this case the total force) is – **F cos 60° = 10 (1/2) = 5 Newtons.** This force must cause the block to accelerate, so

$$F = ma \quad \Rightarrow \quad a = F/m = 5/2 = 2.5\ \text{m/s}^2$$

Think about this solution. It is not very standard, but at least once you have solved something, this helps you see the net force as actually causing the acceleration. The more simple and standard way of solving this problem is using vector components and writing Newton's laws component-wise. Let's see how we can use this approach.

Dynamics:



In the free body diagram, let's resolve each force into horizontal and vertical components. **N** and **mg** only have a vertical component. **F**'s horizontal component is **F cos 60°**. **F**'s vertical component is **F sin 60°**.

Writing the equations:

Net vertical force upwards = N + F sin 60° - mg = m $a_{vertical}$ = 0

Net horizontal force = F cos 60° = 5 Newtons = m $a_{horizontal}$

Mathematics: We have two variables ($a_{horizontal}$ and N) and 2 equations. Solving, we get **N = 11.4 Newtons** and **a = 2.5 m/s²**.

A moment's thought shows that is exactly the same as what we had done earlier, except that this can be done without thinking too hard about what is the direction of the net force etc. Even as you do this, it is good to think in terms of arrows adding up to give a final arrow (even if you don't use it to solve problems). This helps to get a clear picture of what forces to resolve and in what directions and more importantly also gives an overall picture of forces and acceleration.

What happens if **F = 30 N**? Till the writing of the equations, the solution is exactly the same! Solving the equations, we get **N = mg - F sin 60° = 20-25.98 = - 5.98 Newtons**. That means the Normal force is negative! The floor surface is actually pulling the block - but that is impossible. Something is wrong. Our earlier Kinematics analysis tells us what is wrong. We assumed the block moves horizontally. In the second approach (resolving forces), we assumed **$a_{vertical}$ = 0** - this is what is wrong. In the first approach (adding arrows), we assumed that the body only accelerates horizontally and therefore the final resultant force direction is horizontal - that was wrong. The resultant force is at an angle upwards to the horizontal - just the vector sum of **F** and **mg** (**F** is larger than shown) - because N is zero.

Dynamics: The body now has both horizontal and vertical components of acceleration. Since the body just loses contact with the floor, so the Normal force must be zero. So the equations are:

Net vertical force upwards = N + F sin 60° - mg = F sin 60° - mg = 5.98 Newtons = m $a_{vertical}$

Net horizontal force = F cos 60° = 5 Newtons = m $a_{horizontal}$

Mathematics: Solving the equations, we get **$a_{vertical}$ = 2.99 m/s²** and **$a_{horizontal}$ = 2.5m/s²**.

Earth, Weight and Gravity

We know the earth pulls all objects towards itself. Newton's gravitation law says that this kind of gravitational pull is true for any two particles - and the force between two masses **m** and **M** which are separated by a distance **r**, is given by:

$$F = \frac{G\,m\,M}{r^2}$$

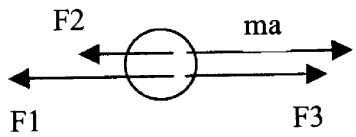
The force on both masses is exactly the same - as Newton's third law says it should be. You pull the earth with the same force as the earth pulls you. Of course the earth's pull on you makes you fall quite a bit - and your pulls shakes the earth very little as the earth is so massive.

This force on you changes with the distance from the center of the earth. But we are already 6400 km from the center of the earth (on the surface). Moving a few kilometers up and down won't change this distance much. So on the surface (and even on airplanes - which may move up a few hundred kilometers), **r** is almost constant. **M** = mass of the earth is a constant and **G** is a universal constant. This means the earth's pull on an object near the surface is basically a constant times mass of the object. But force **F** on a mass **m** must cause an acceleration = **F/m**. So this constant must be the acceleration of objects near the earth's surface that have no other forces on them. What Newton's gravitational law tells us is that acceleration of all objects falling freely near the earth's surface is the same = $g = G\,M_{earth}/R^2_{earth}$. Galileo experimentally observed this fact that *all objects fall towards the earth with the same acceleration irrespective of their masses*. This is because *always* acceleration is inversely proportional to the mass and *in gravity* force is proportional to mass! So the mass cancels out and all objects have the same acceleration. When we talk of weight and force due to the earth on a mass, we will use **mg** - mass times the acceleration due to gravity. Force due to earth's pull is not **mg** at all heights. Once you are very far away, the force (and therefore your weight) decreases. Using the general gravitational law, you can derive how planets move, and how long the moon should take to go around the earth, etc. But this book just looks at the basics and so we will not bother about the more general gravitation force law. We will remain close to the earth's surface and so gravitational force is just **mg**.

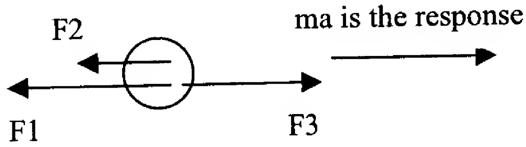
Note: **g does not cause mg**. Sometimes students think "g is the acceleration due to gravity. A body with acceleration **a** must cause a force = **ma** on the body. So the force must be **mg**." This reasoning is wrong. **g** *does not cause* **mg**. **a** does not cause **ma**. Force causes acceleration - not the other way around. **mg** *causes* **g**. There is **g** because the earth pulls with **mg**. **mg** is just a convenient way of representing the earth's pull. If you are sitting on a chair, you are not accelerating with **g**. But there is still a force **mg** on you. The earth always pulls you - irrespective of what you are doing. We say **mg** only because otherwise we have to say GMm/r^2 and that is very tiring and long! But that *doesn't* mean acceleration due to gravity *causes* the force.

There is a more dangerous problem. Because we use **mg** as a force (and also because **F = ma** makes **ma** sound like a force), many students think of **ma** as a force. So they draw a free body

diagram with **ma** drawn as one of the forces. See the two figures. The one on the left suggests **F3 + ma = F1 + F2**. The correct free body diagram is shown on the right and says **F3 - F1 - F2 = ma. Never draw ma on the free body - it is not a force.** The one exception is **mg** - again this is just a way of writing earth's force and **not** m times the acceleration of the body.



Wrong!

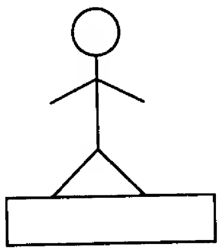


Right!

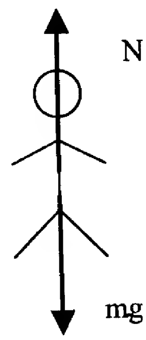
This pull on us by the earth is what we call our weight. Weight is not mass - weight is a force. When we say these potatoes weigh 5 kg, what we mean is that if you measure the pull by the earth on these potatoes and measure the pull by the earth on a mass of 5 kg - it will be the same. Unlike mass, weight has direction. You weigh downwards - you don't mass downward!

Some of you may have gone on roller-coaster rides, or on lifts. When you come down, you suddenly feel a bit lighter. This is the same feeling you get when you fall down from a high place. What really is happening is that you are accelerating towards the earth and so no longer "feel your weight". Let's try to understand this. You are normally used to your weight. But what is this that you are used to? When you stand on the floor - you are not falling or accelerating towards the earth. But the earth still pulls you down. You don't fall because the surface holds you up with a force - Normal force in many cases. What you feel is really this force by the ground on your legs or on your butt!

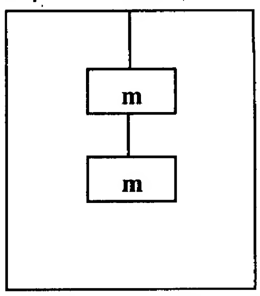
When you stand on a weighing machine to measure your weight, what your really measure is the force with which you push the surface of the weighing machine (or equivalently the force with which the surface of the weighing machine pushes or holds you up). This is what we call the sensation of weight - or apparent weight. When you stand on a weighing machine and you are not accelerating towards the earth, the total force on you must be zero.



Let's draw a free body diagram for the person as shown on the right. There is a downward force exerted by the earth – weight – **mg**. The surface prevents you from falling down by exerting a Normal force upwards (actually this force is on the legs, but you can shift the force around). There are only these two forces on the body. The body does not move (has no acceleration). So **N - mg = m** $\mathbf{a_{vertical}}$ **= 0**. So N = **mg**. This is the force with which the weighing machine surface pushes you and you push the weighing machine surface. This force is what the spring shows.



Now, let's say you are in a lift that is moving up with an acceleration **a**. What are the forces on you? It is still exactly the same two forces **N** and **mg**. But together they make you move up with an acceleration **a**. This means **N - mg** must cause this acceleration => **N-mg = ma** and therefore **N = m(g+a)**. So if you were standing on a weighing machine you will push it with a force **N = mg+ma**. Since the weighing machine is calibrated to show force/g, it will show your mass (actually it only shows weight) as **m(1+a/g)** kg. So you will seem to weigh more. Your sensation in your legs of course will also show that you are being pulled down more. Actually that's not true - rather you are being pushed up more by the floor. And that is what you feel as your weight. Carefully imagine this situation and think about it. Earth's pull on you (the real weight) always remains the same. What you feel is actually the normal force - and this changes if you are accelerating.

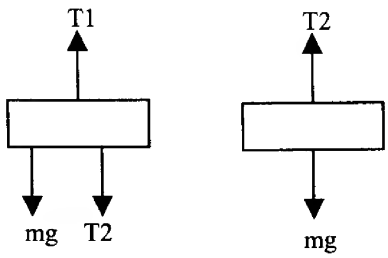


Problem 4: In the figure shown, a lift is accelerating downwards with an acceleration **g/3 m/s²**. What is the tension in the top and the bottom ropes - assume ropes are massless and both the blocks have a mass **m**. What is the answer if the lift accelerates upwards with **g/3**?

Solution:

Kinematics: This part is quite simple - the two blocks move together down with **g/3** (or up with **g/3** in the second part of the problem).

Dynamics and Free body Diagrams: Let's say the tension in the top string is T_1 and the tension in the bottom string is T_2. Then we have the following free bodies for the two masses (the top one on the left and the bottom on the right).



From these two diagrams it is very clear that the two equations are:

Net force on top block = $mg + T_2 - T_1 = ma = mg/3$
Net force on bottom block = $mg - T_2 = ma = mg/3$

Once you draw the free-body diagram (only forces - no accelerations, no **ma**), the equations are very simple to write. There are two variables T_1 and T_2 and 2 equations and they can be solved. If the lift is accelerating up instead of down, again the free body is the same and so is the net force - what changes is only the acceleration of the body.

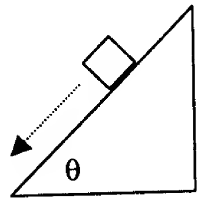
So for the second case the equations are:

Net force on top block = $mg + T_2 - T_1 = ma = -mg/3$
Net force on bottom block = $mg - T_2 = ma = -mg/3$

Mathematics: Solving the equations in the first case and in the second case, we get:

Case I: Lift accelerates down with **g/3**: **T2 = 2mg/3** **T1 = 4mg/3**
Case II: Lift accelerates up with **g/3**: **T2 = 4mg/3** **T1 = 8mg/3**

Problem 5: A body of mass **m** is on a fixed inclined plane of angle **θ**. All surfaces are frictionless and smooth. What is the acceleration of the body and how much does the body push the incline?

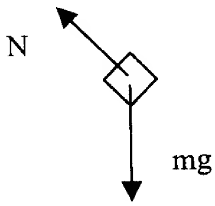


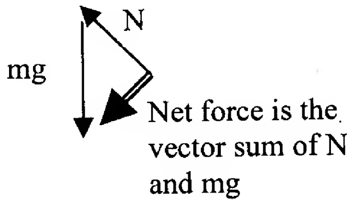
Solution:

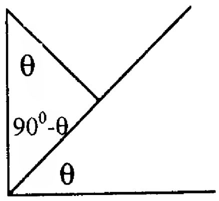
Kinematics: Looking at the figure and with our intuitive feel for the problem, we know that the mass will have to move down the incline - along the incline. So its acceleration will also be along the incline. Call it a. This means it has zero acceleration perpendicular to the incline.

What knowledge does kinematics really give us? It does not give us the magnitude of acceleration. But in this problem it has given us the direction. So we know the magnitude in the perpendicular direction (zero!). Acceleration (in this planar problem) has two unknowns. Kinematics intuition gives us one equation (or eliminates one unknown).

Dynamics: As always draw the free body diagram. The force by the inclined plane is Normal to its surface because it is frictionless (so cannot give force in any other direction, but normal!)







The first diagram above show the free body with forces on it. There are just two forces on the body. Together they must make the body move down the incline. **mg** is a known force. **N** is an unknown force. We know the sum of the two vectors must add up to a final net force vector arrow in the direction of the acceleration. So **N** must be such that it makes this happen. This is shown in the second figure. This triangle solves the problem. But we need to know the angle between **mg** and **N**. **N** is perpendicular to the plane, **mg** is vertical and the plane is at an angle q to the horizontal. So what is the angle between **N** and **mg**? The third figure shows it must again be **θ**. (*Avoid short cuts at this point - one extra diagram doesn't cost you anything - but helps you avoid mistakes. Always draw a geometry diagram to find the angle.*)

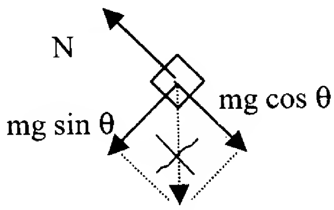
With this information and just from the second diagram (a right triangle) we can say **N = mg cosθ**. And the net force must be **mg sinθ**. Therefore **acceleration must be g sinθ**.

It looks like the constraint that the body moves on the plane is actually a constraint on N! How does N know exactly what to be for this to happen? If you asked this question, then you are thinking in the right direction. Let's argue from the inclined plane's perspective. I am the plane. First I don't exert any force on the block - why should I ? It doesn't trouble me. So there is only mg on the block. So the block starts moving downward - that means it tries to pierce me. So I push the block away to stop it piercing me. If I push it too much, it moves so

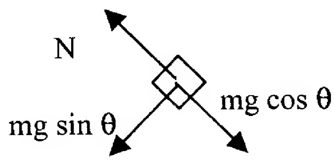
far away from me that it is no longer trying to pierce me. So I stop pushing it. If it don't push it enough, it will continue moving in my direction and will pierce me. So I must push it the right amount to neither allow it to pierce me, nor push it away from me so much that it moves away and I cannot exert a force on it any longer. This is the way the plane figures out what the exact force on the body should be - the force the plane exerts is the least amount of force it needs to exert to keep the body from piercing the plane. The plane doesn't unnecessarily exert more force, when less will do! This is the way the plane makes the body move along the incline. Normal force and **mg** together cause the body's inclined acceleration. But in solving problems, we often need to start from the acceleration and work backwards to find what the force must be to ensure such an acceleration.

The above is a direct vector approach to solving the problem. Instead let's now do the problem using vector components. This is very straight forward - but it removes the reasoning behind why things work. It is important to learn both approaches.

Components approach: We know the acceleration is in the direction of the plane (perpendicular to **N**). Resolve the forces in the direction of the plane and perpendicular to it. The first free body diagram above is drawn below in terms of the resolved forces.



This is **not** a good free body diagram. This is just to show you how to resolve mg.



This **is** a good free body diagram. It only has the 3 forces – and nothing else.

The first figure above shows how **mg** is resolved into two components - along and perpendicular to the plane. To find the components, just project the original vector in the direction you want the component in - it is like shining light and finding where the shadow will be. Using simple trigonometry you can see the angle between **mg** and plane is **90°-θ**. So the component along the plane is **mg cos(90°-θ) = mg sinθ**. Similarly the angle between **mg** and the perpendicular is θ and so the component is **mg cos θ**. *(See the note on vectors at the end of this chapter to find out how to take components.)*

Note: Once you resolve a force - only the components remain. You cannot keep both the original force and the components - keep only one of these. As an interim step you may use the free body to resolve forces - scratch out the original force. Make a neat final free body diagram with just the forces **or** their components **(not both)**.

Writing the equations:

Net force along the incline = mg sin θ = ma

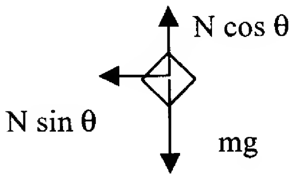
Net force perpendicular to the incline = N - mg cos θ = m $\mathbf{a_{\text{perpendicular to incline}}}$ **= 0**

We have 2 equations and two unknowns **N** and **a**. We can solve them.

Mathematics: Solving the equations, we get **N = mg cos θ** and **a = g sin θ**. And **N** is the amount by which the block pushes the inclined plane. This is the force of interaction between the block and incline. If there was friction, both **N** and friction would together be the force of interaction.

S: **Why did we resolve mg? Why cannot we resolve N instead of mg ?**

T: This really means you want to resolve **N** horizontally and vertically instead of resolving **mg**. In principle this is not wrong. Let's do it.



S: Look at the free body diagram. So now **N cos θ = mg**. So **N = mg cos θ** and **N sin θ = ma** and so **a = g cosθ sinθ**. This is different from the earlier answer. Why is this not right?

T: Till the free body it is all correct! You can really resolve the forces the way want. Only thing is your must do it correctly.

S: Then why is my answer wrong?

T: Because you did not write Newton's laws! You jumped to conclusion. Newton never said force in one direction always cancels with force in another direction. Why did you make **N cosθ = mg ?**

S: But you also said **mg cosθ = N.**

T: Yes. But before that I wrote Newton's law in the normal direction. Because the acceleration is zero in that direction, I got that the two forces are equal and opposite. In your case, the body is coming down - so it has an acceleration downwards as well! So **mg - Ncosθ** is not zero.

S: Then what is it?

T: Let's do the problem as you started out. My kinematics tells me that a has to be along the incline. So resolving a, we get that **the object has an acceleration a cosθ horizontally to the left and a sinθ vertically downward.** So Newton's laws in the horizontal and vertical directions are:

$$mg - N\cos\theta = m\,a\sin\theta$$
$$N\sin\theta = a\cos\theta$$

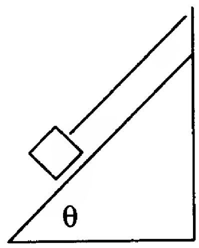
Again we have 2 equations and 2 variables. Solving these (use $\cos^2\theta + \sin^2\theta = 1$), we get: **a = g sinθ and N = mg cosθ.** This is the same answer as before.

S: Oh! So the mistake was in assuming two resolved forces in opposite directions - here **mg** and **N** $\cos\theta$ - always cancel.

T: Yes - they only cancel if there is no acceleration in that direction.

S: In a general problem how do we know what direction to resolve forces in and which force to resolve and which not to resolve?

T: The kinematics will give you a hint - usually the direction of acceleration and perpendicular are good directions. But again not always. Some directions make the problem easier to solve. But all directions are correct. ALL FORCES MUST BE RESOLVED IN THE TWO DIRECTIONS YOU CHOOSE. It may happen that some forces are already parallel or perpendicular (like in our earlier case **N** was already resolved) - but that is coincidence. And in each direction **sum of all the resolved forces should give you ma** in that direction. It **may** happen that **sometimes a** is zero and so this sum of forces is zero. But that's a special case - it will come out of Newton's laws. Don't start with that as an assumption. Always derive it as a consequence.

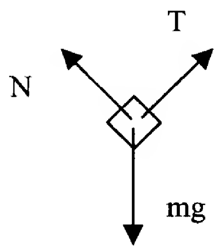


Problem 6: In Problem 5, I tie the block to the top of the incline with a string. What will be the normal force on the block now? If instead of the string, I push horizontally and stop the block from moving, what is the normal force then?

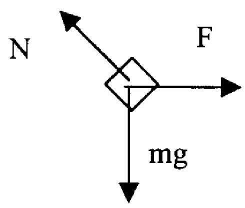
Solution:

Kinematics: This is very simple. The body does not move. **a = 0** !

Dynamics: Let's draw the free body diagram in both cases.



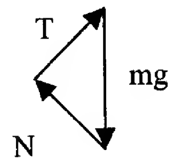
Case I
String holding the body



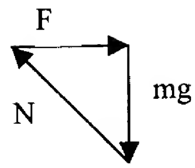
Case II: Horizontal Force
(Always draw forces outwards from the body)

In both cases the sum of forces must be equal to **ma** which is zero. So if you draw the vector

(arrow) diagrams for the forces, we get



Case I



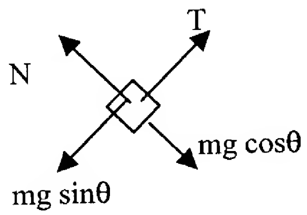
Case II

See these diagrams. In Case I, **mg** is the hypotenuse and so **N = mg cosθ** and **T = mg sinθ**. In Case II, **N** is the hypotenuse, and so **N = √[(mg)² + F²]** and at an angle of **$\tan^{-1}$(F/mg)** to the vertical. But we know this angle has to be **θ**. So **F** has to be **mg tanθ**. Therefore **N = mg/cosθ**. Instead of writing the magnitude we can also write **N** as

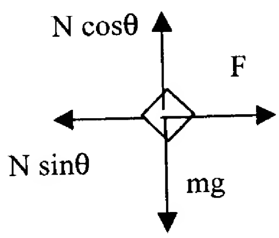
$$\mathbf{N} = -F\,\mathbf{i} + mg\,\mathbf{j} = -mg\tan\theta\,\mathbf{i} + mg\,\mathbf{j}.$$

*Can you try and imagine why **N** is larger in the second case and not the first case? Think about what causes **N** in the first place?*

Instead of vector addition, if we use components, we have to following diagrams. (We use plane and normal as the axis in Case I and horizontal and vertical in Case II because that means we have to only resolve one force in each case.)



Case I



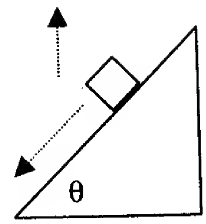
Case II

Writing the Equations:

Case I:	**N - mg cosθ = 0**	**mg sinθ - T = 0**
Case II:	**N sinθ - F = 0**	**mg - N cosθ = 0**

Mathematics: Solving the above equations we get the same answers as we got earlier.

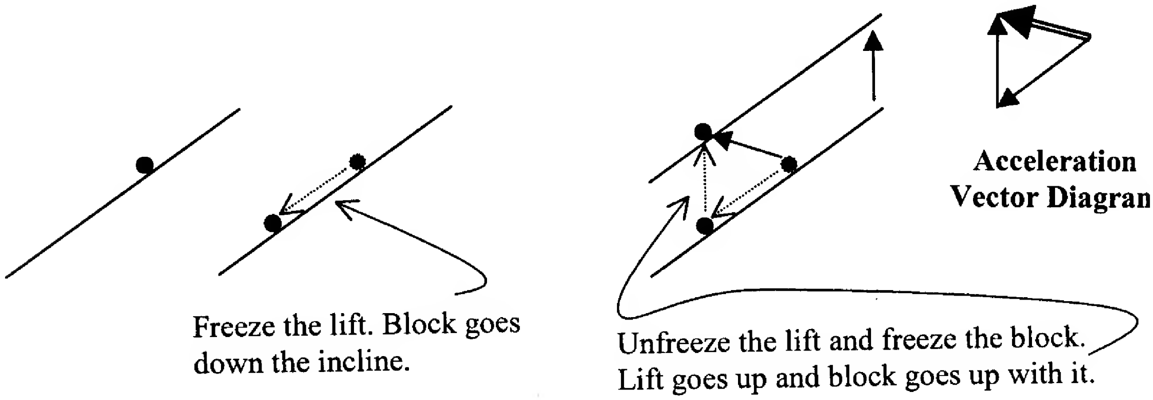
Problem 7: What happens if in Problem 6, the inclined plane is on a lift and the lift is actually accelerating up with an acceleration **A**? Find the actual acceleration of the block and the normal force exerted by the plane on it.



Solution:

Kinematics: This is the tricky part here! The rest is the same as before. The inclined plane goes up and the block goes down the incline. How does the block effectively move?

Let's do this in three steps.



The figures above show that the actual acceleration of the block has two parts - one is the acceleration along the incline and the other is the acceleration upwards of **A**. This is the kinematics of the problem. In the figure it is shown as if the block actually moves upwards and left. But this need not be the case. If **A** is very large, this will be the case. On the other hand if A is small, but the acceleration down the incline (say **a**) is large, then it will move net downward and left.

S: But now we don't know the magnitude of the acceleration down the incline. So we don't know the magnitude of the actual acceleration. And we don't know the direction of this acceleration. So what really is the information our kinematics has told us?

T: Good question. The kinematics tells us that the actual acceleration can be written as the sum of two arrows - one at an angle θ to the horizontal (along the incline) and the other **A** vertically upwards.

S: But any vector can be written as the sum of two vectors. This doesn't give us any new information.

T: Right. In our usual 2D problems, an unknown vector is equivalent to two variables - 2 unknown numbers. Magnitude and angle or x and y components. When for example you write **a** = x **i** + y **j**, there are two unknowns - x and y. This means you know the vector **a** is the sum of two vectors - both of unknown magnitude. But suppose I say magnitude of the **j** direction vector is known, that means you know y. So there is only one unknown. In this case it is easy to see. But in our problem, we know the magnitude of the vertical vector - it is

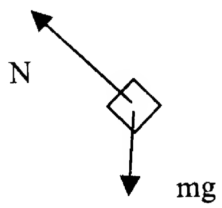
A. So the only unknown is the magnitude of the inclined acceleration vector **a**. So only one number is unknown - a. This is what our kinematics has provided us with. It has eliminated one variable, or equivalently it has provided us with one equation.

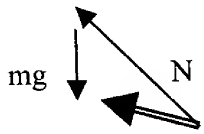
Proceeding with our kinematics analysis, the actual acceleration can be written as

$$\mathbf{a}_{\text{actual}} = -a\cos\theta\,\mathbf{i} + (-a\sin\theta + A)\,\mathbf{j}$$

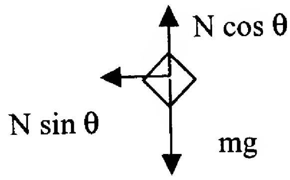
where **i** and **j** are the unit vectors towards the right and upwards respectively. This means the body has a leftward acceleration of $a\cos\theta$ and an upward acceleration of $(-a\sin\theta + A)$.

Dynamics: Even in this complicated problem, there are only two forces! The free body diagram is as simple as earlier. In fact this is the power of the free body approach.





Net force is the vector sum of N and mg



The second figure shows the sum of the forces - this must be in the direction of the acceleration. But here it is not easy to calculate with this arrow diagram. So let's use the component approach. Since the kinematics gave us accelerations in horizontal and vertical directions, we resolve all the forces in these two directions. (We could have resolved **A** along and normal to the plane in the kinematics section - then we would have chosen these as the directions to resolve the forces).

Writing the equations:

Net force upwards = N cos θ - mg = m × acceleration upwards = m (-a sinθ + A)

Net forward leftwards = N sin θ = m × acceleration leftwards = ma cos θ

2 equations and 2 variables - **N** and **a**. So we can solve these.

Mathematics: Solving the equations we get, **a = (A+g) sinθ** and **N = m (A+g) cosθ**. Since the actual acceleration was asked the answer is:

$$\mathbf{a}_{\text{actual}} = -(A+g)\sin\theta\cos\theta\,\mathbf{i} + (-(A+g)\sin^2\theta + A)\,\mathbf{j}$$

What an amazing number of situations are caused by just two forces - **mg** and **N** - acting on a body! The student will notice that we started with one problem and are slowly modifying it into another problem and yet another problem. This is the best way to learn Physics. Learning

to do 10 completely new problems teaches you very little. But creating 10 problems that are slightly different helps you get at the basic principles and grasp them well. Whenever you see problems - try to do this and create your own modifications. You will find that this really helps. Here are two more modifications you can try.

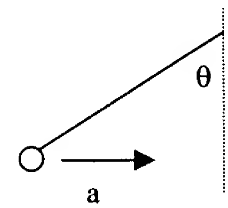
Problem 8: What happens in problem 7 if the block is tied by a rope to the top of the inclined plane. (Easier than problem 7).

Problem 9: Instead of fixing the inclined plane on a lift, you place it on a smooth frictionless surface. The inclined plane has a mass **M** and the block has a mass **m**. Try to solve this problem and find what the acceleration of the inclined plane should be. (This is a reasonably tough problem - but with some effort you can do it.) *Hint: Now the inclined plane will move towards the right and the block will slide on the incline. The real acceleration of the block will no longer be along the incline. Do the kinematics - that's the tricky part. The rest is easy.*

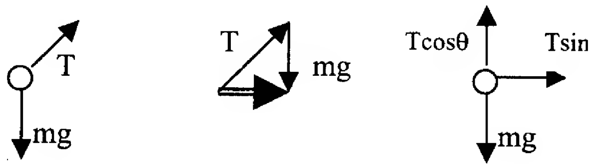
Problem 10: A pendulum is hanging from a train which has an acceleration **a**. What is the angle at which the rope is inclined to the vertical?

Solution:

Kinematics: We assume that the pendulum bob is stationary relative to the train and all the initial oscillations have died down. That means the bob is moving along with the train and has a horizontal acceleration **a**.



Dynamics: We start with the free body diagram for the bob. Assume the rope is tilted at an angle **θ** with the vertical. There are just forces on the bob: **mg** and tension. These together give the bob a horizontal acceleration. The third diagram on the right shows the resolution of forces.



Writing the equations: Since the acceleration is horizontal, we get the following equations:

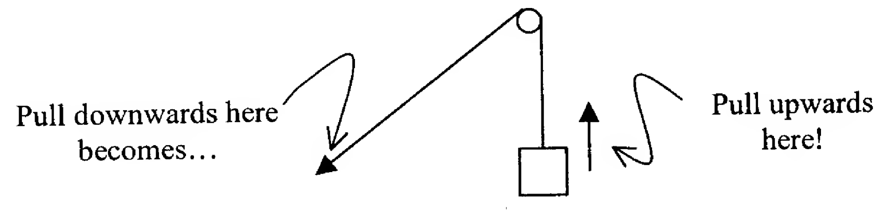
$$\mathbf{T\cos\theta - mg = ma_{vertical} = 0}$$

$$\mathbf{T\sin\theta = ma}$$

Mathematics: Solving these equations, we get $\mathbf{\tan\theta = a/g}$ or $\mathbf{\theta = \tan^{-1}(a/g)}$.

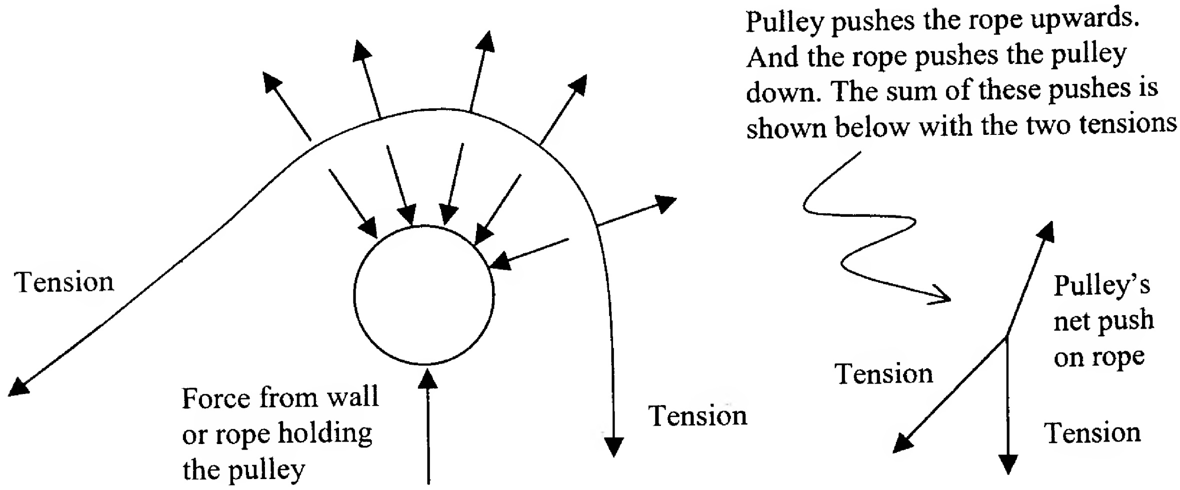
Pulleys and Round Pegs

The next object we are going to see in this book is the pulley. Pulleys are familiar objects to many. A rope can pass over a pulley. When you pull one end of the rope, the other end of the rope gets pulled as well. Therefore by pulling down, you can actually lift something up.



How does the pulley convert a downward force at one end into an upward force at the other end?

By exerting a force on the rope! The pulley holds up the rope by exerting a force on it. Let's look at the free body diagram of the pulley and the rope.



In the figure you can see how the pulley pushes the rope at various points. The sum of these forces is what keeps the rope up, turns the rope around still retaining its tension. On the right is shown an arrow diagram with the two tensions and the net force by the pulley on the rope. It is because of this push upwards that both parts of the rope can pull downwards.

S: But why do we need a pulley for this? Won't this work even if I just catch a part of the rope and pull it upwards? We will get the same diagram even in this case - won't we?

T: Yes - the arguments will work even if you catch a rope and pull it upwards yourself. But the difference between a pulley and your pull is in this fact - the tensions on both sides of the

rope are the same (at least for a good frictionless pulley). Let's first think about your catching the rope tightly. If I pull one of the ends of the ropes, is it necessary that the other rope have any tension?

S: No. It can just hang.

T: Right. In fact often this is what happens when we pull with a rope, we hold the rope in the middle and pull and one side of the rope with which we are pulling is taut while the other side of the rope is limp. Similarly you can imagine someone else pulling the other side of the rope. Now both sides have tension - but different tensions. Your force balances the sum of these two forces on the two parts of the rope. In fact for all practical purpose this is just like having two different ropes held together by you.

S: Yes - but how is the pulley any different? Why do you say the pulley makes the tension the same?

T: Look at the force by the pulley on the rope. It is always perpendicular to the direction of the rope. The pulley is only pushing the rope away - at no point is it increasing or decreasing the tension. Tension comes about because we try to stretch a rope. If there is no attempt to stretch it, there is no tension. Turning a rope is not stretching. Only if the pulley has friction, it can pull the rope *along the rope*. Just try to imagine this situation and think about it. This will give you a feel for it.

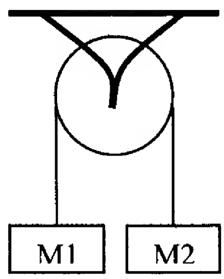
S: Oh! I see! When I hold and pull the rope, I am pulling along the rope as well - so I can change the tension substantially. This is again only because there is a lot of friction between my hand and the rope.

T: Yes. That's why to hold slippery ropes you make a knot or make a circle within which you can hold it. Coming back to the pulley, we can see that the pulley exerts forces normal to itself and to the rope and so only turns the rope and does not change its tension. This is very important. This way the rope is just one rope - pull at one end means pull at the other end as well. Of course if the pulley has friction, we cannot assume this. There are ways of calculating what the tension will be if there is friction in the pulley, but here we won't get into that. (*For those interested - try to find the normal force at each point on the pulley and the corresponding friction force assuming the rope slips*).

In most of our problems, we will also assume that the pulley is massless and that way it does not matter whether the pulley is rotating or not. But if there is no friction, it really does not matter whether the pulley has mass or is rotating. But real pulleys do have friction! So if the mass of the pulley is small, then it rotates with the rope and makes sure there is no slipping of the rope on the pulley. This allows the friction to be small and therefore we can assume the tension to be the same throughout (if the rope is also massless).

Why is a pulley so useful? If all directions for exerting a force equivalent then, we wouldn't need pulleys so much. But often as in pulling water out of a well, pushing down is easier than pulling up. So pulley helps convert a force in one direction into a force in another direction. But this is not all. Pulleys can also produce a higher force or a lower force. This is

useful in lifting very heavy object and in precision adjustments respectively. The problems below illustrate these ideas.



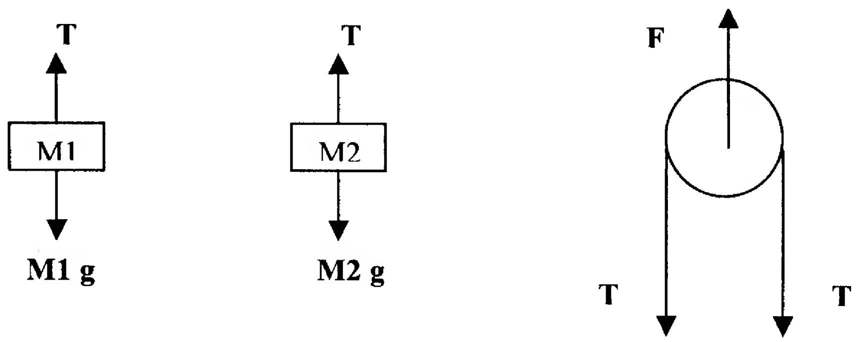
Problem 11: In the figure shown, the two masses are hanging from a smooth massless rope moving on a smooth pulley. What is their acceleration and what are the forces on the pulley?

Solution:

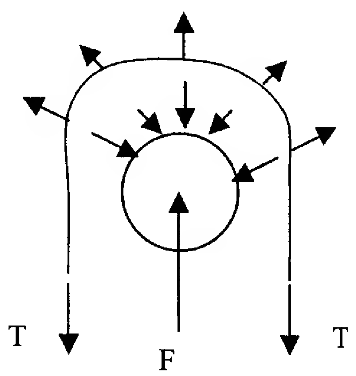
Kinematics: If M_1 goes down by some amount, M_2 must go up by the same amount. (Because rope's length does not change). This means the acceleration magnitude of M_1 and M_2 must be the same. (Why? Think. If the displacement is the same, you can say acceleration is the same, but if the acceleration is the same, you cannot say the displacement is the same.)

So let's say **a** is the acceleration of M_1 downwards. This means M_2's acceleration is **a** upwards.

Dynamics: The tension in the rope is the same throughout because the pulley is frictionless and rope massless. So drawing the free body diagrams, we have:



To find the acceleration, we don't need the free body diagram for the pulley. The first two diagrams are enough. But to find the forces on the pulley we need its free body diagram. We have actually drawn the free body diagram for the pulley and the part of the rope which is on the pulley together. If we want the free body for the pulley alone, we should draw the diagram as shown on the right. But the free body for the rope on the figure shows that the sum of all the small forces is the same as the two tensions. (*Can you see why?*) Therefore we can replace all the small pushes between different parts of the rope and the pulley by just these two tension forces. The force by the wall on the pulley is represented by **F**.



Writing the equations: Each free body gives us one equation:

$$M_1 g - T = M_1 a \qquad T - M_2 g = M_2 a \qquad 2T - F = 0$$

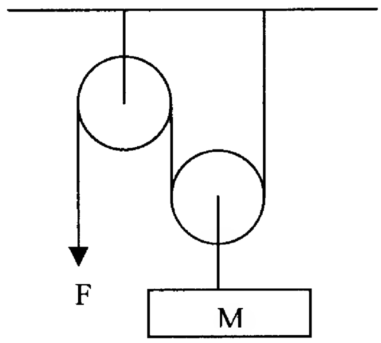
We have three equations and three variables. The first two gives us the acceleration and the Tension. The last one gives us the wall's force on the pulley.

Mathematics: Solving the equations we get, $a = g(M_1 - M_2)/(M_1 + M_2)$ and $T = 2gM_1M_2/(M_1+M_2)$ and $F = 2T$. If the masses were the same, the acceleration will be zero as we expect. The wall has to exert twice the tension to keep the pulley in place.

◆————————◆

In the above problem, if instead of using M_1 we just tried to pull M_2 up slowly by pulling the rope, then we would have had to exert a force of M_2g. *(You can draw a diagram and show this must be the case. Since the motion is slow, the acceleration of M_2 is zero.)* But to pull up M_2 ordinarily we would have still used a force of M_2g. It looks as if pulleys don't help us reduce the amount of force. That's because we are using just one pulley and in a very simple way. Below is a problem that shows what you can do with two pulleys.

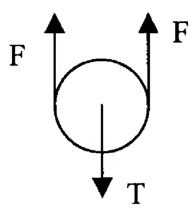
Problem 12: In the figure are shown two pulleys with a rope passing through them as shown. A mass **M** is hanging from the second pulley and you want to pull the mass up by exerting a constant force **F** on the rope as shown. What is the minimum force you need to exert for the Mass to move up? Assume all pulleys and ropes are massless and frictionless.

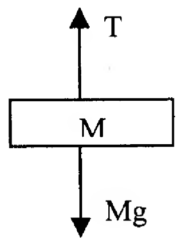


Solution:

Kinematics: You want the mass to move up. But you also want the minimum force. That means the mass **M** must move very slowly - it is *just* moving up. This means the acceleration of the Mass is zero.

Dynamics: The tension throughout the main rope is **F**. Below is a free body diagram for second pulley and the mass. (One could have also drawn a joint free body diagram for the two together).





Writing the equations: The Pulley's mass is zero. So we get:

$$2F - T = 0 \qquad\qquad T - Mg = Ma = M \times 0 = 0$$

Mathematics: Solving the equations, we get $T = Mg$ and $F = Mg/2$.

So you have to exert a force of only half the weight to lift the weight up. This is one of the reasons why pulleys are used - to reduce the force that we need to exert. Of course this does not mean we can manage with a smaller force everywhere. The roof has to exert a upward force on the whole system and this is more than **Mg** (= **3Mg/2**). But since we don't care about how difficult it is for the roof, it at least eases our life! But don't think we have got something for free. If you think about it, you will see that while your hand goes down by some distance, the block goes up only by half the distance. So you have managed to exert half the force, but now you need to move double the distance. In older textbooks they call this the principle of pulleys and levers. The product of the force exerted and the distance moved is the same. This product is called work. So you do the same work - but now you can do it by exerting a smaller force over a longer distance. The energy spent by you is the same. So you don't gain much in terms of energy - but at least you don't hurt yourself trying to pull too hard! These ideas of work and energy are very important - but we will reserve it for another book!

◆————————◆

Try to work out modifications of the above problems with more pulleys and find the accelerations of objects. Here are a few problems for you to try. You can create more problems with 3-4 pulleys and see what you get when you solve them.

Problem 13: In problem 12, instead of pulling with a force **F**, I hang a mass M_1 from the rope. What will be the acceleration of the block **M**? Is it still true that M_1g times the distance moved by M_1 is equal to **Mg** times the distance moved by **M**? Why not? What happened to the energy? What if M_1 is half of **M**? Why does it work in this case?

Problem 14: The pulley in Problem 11 is actually on top of a fixed right angled inclined plane with angle of incline θ. One of the masses m_1 lies on the incline and the other mass m_2 is hanging from the pulley straight down. Find the acceleration of the masses assuming the incline is frictionless.

Problem 15: In problem 14 what happens if the inclined plane itself has a mass **M** and can move without friction on the floor? Find the acceleration of the inclined plane.

We can go on like this with more and more complex problems. Many books do this and there are a large number of very interesting problems that you can solve. But this is not a problem book. I introduced some problem solving techniques just to help you get a feel for how we use Newton's laws. Let's now get back to discussing the meaning of Newton's laws.

What do Newton's Laws Really Mean?

T: Newton's first law says what an object does when there is no force - *it moves uniformly in a straight line.* Newton's second law says what a force does to an object - *accelerates it.* Since we are dealing so much with forces, it seems relevant to ask what exactly is this thing that we call force? (*I know it is a bit late to ask this question - but better late than never!*)

S: Force is a push or pull. It is how one object interacts with another - stops it, pushes it, pulls it, etc.

T: Yes. This is the definition we used earlier. But let's say I want to measure the quantity of force exerted. How do I do that? Or at least is it possible to find out when there is no force on an object?

S: When the object is moving uniformly, there is no force on it.

T: But this is circular. If there is no force the object moves uniformly. How do you know there is no force? Because the object is moving uniformly. This kind of argument is not science. So what does Newton's laws really mean? And what is a good definition of force?

S: Can't we just say **F=ma** is the definition of force?

T: Yes, we can and many people use this as the definition for force. So to measure the amount of force, you just let it accelerate a body and measure its acceleration. (Length and accelerations are more easily measured). So this works as a good measuring technique and therefore a good definition for force.

S: So that's solved it. **F=ma** is the definition for force and Newton's laws make sense.

T: Not so fast! If **F=ma** is just a definition, then what is so great about it? Definitions are human creations - they are not physical statements about the world. We can create other definitions. Why then should we use **F=ma**. Maybe **F=mv** will also work equally well?

S: You mean if **F=ma** is a definition, it is not a law anymore? It has no physical meaning?

T: Yes. Something is wrong with this. Clearly, **F=ma** is a very useful idea - as we just saw. It helps us solve a number of problems. So if it is a mere definition, the question is why does it work so well? Will other definitions work as well?

S: You mean instead of **F=ma**, we could have started with **F=mv** as the definition and things wouldn't have been quite as simple?

T: Exactly. **F=ma** is a special definition. Take for example weight. Weight is a force (**mg**). If instead force was defined as **mv**, then each body will have a different weight as it is falling down and as its speed increases. The formula for weight that was so simple in the earlier case, now would be a complicated function of time. Other forces will turn out to be even more complicated. It is only with **F=ma**, that **F** turns out to be very simple.

The real meaning of Newton's second law lies in the idea that if you define force to be mass times acceleration, then you will find simple force laws for interaction of objects. **F= ma** is

not a complete statement. You need force laws to complete the equation. You need formulas for **F**. Newton's genius lay in discovering that if you define **F** as **ma** and then search for laws for this **F**, you will find simple laws. Basically you are looking at patterns for **ma** and Newton's law says that **you will be able to find these patterns** and they will be simple.

S: Oh! So that's what Newton's second law means. What does the first law mean? Anyway why do we need the first law? **F = ma**. So if **F** is zero, acceleration has to be zero and that means constant velocity. **Isn't the first law contained in the second law?**

T: Let's look at the nature of the first law. Earlier people used to think that rest was a special state - the natural state for objects. Things at rest were seen as doing what is normally expected of them. When there is no influence on a body, people thought it would be at rest. The first law says that there is nothing special about rest. Things that are moving uniformly also remain in uniform motion forever. When there is no influence on a body, it can either be at rest or in uniform motion. This led to the understanding that rest seemed special earlier only because we are used to things on the earth stopping because of friction.

Newton's first law does not say all motion is natural. It says only rest and uniform motion is natural. Why is uniform motion so special? Is it really that special? Why isn't it the case that objects accelerating continue to accelerate forever with no force on them? The answer from Newton's first law seems to be "yes - uniform motion is special."

One interesting thing about forces is that it has a material origin. Force on an object is caused by other objects around it. If there are no objects nearby, there is no force on you. Let's call such an object with no other objects nearby, a *free* object. Imagine such a free object in deep space. Newton's first law says this object must move with constant velocity. But there is a catch! Description of Motion is relative. The question is who sees it moving with constant velocity? Not everyone!

In discussing motion, we have so far not looked at the observer. But an observer is critical to describing motion. Different observers see different motions for the same object. If you are traveling in a train and you throw a ball up - to you the ball just goes up and comes down. But to a fellow on the ground the ball also moves along with the train - the ball goes in a parabola.

So when we say a body moves uniformly in a straight line, the question to ask is "Who **sees** the body moving uniformly in a straight line?" I am an observer. Let's suppose that I see the object moving uniformly. Then if you are in a rocket that is accelerating with respect to me, then you will see this *free* object accelerating. But there is nothing near the object that can exert a force. That means you will say that the object is accelerating without any force. That means you see Newton's First Law violated.

S: But that's not right. If you are in a rocket, you should not talk of how the object is moving. People who are moving cannot talk of how other objects are moving. Only people at rest can do this.

T: What do you mean rest? Rest is itself a relative idea. Rest with respect to whom? If I think I am at rest and Newton's laws works well for me, then others moving at uniform velocities

with respect to me can also talk Newton's laws and they will work well for them as well.

S: So are you saying Newton's laws will work for anyone who is in motion as well?

T: No. It will only work for all those who are moving uniformly with respect to me - those observers who are not accelerating with respect to me. That is because Newton's laws deal with accelerations - they don't care what the velocity is as long as it is constant.

S: So Newton's laws work only for one class of observers?

T: Yes - it works only for a special set of observers. These observers are all either at rest or moving uniformly with respect to each other. These special observers are called *Inertial Observers*. Only Inertial observers see a free object in deep space move with uniform velocity.

S: But how do I know I am an inertial observer?

T: That's a very important question. You are an inertial observer if you see free objects in deep space moving with uniform velocity.

S: But that's circular! That's saying nothing really. It is just saying, "if you see something moving at uniform velocity, you will see it moving at uniform velocity". That has no information content!

T: No - the information content lies in the fact that you can check whether I am inertial using one free object. And then this inertial observer will work for all other free objects. So this is not a circular definition. If you see one free object in deep space moving with uniform velocity, you are inertial and therefore will see all other free objects moving with uniform velocity.

This is the basic point of Newton's first law. It states that **there exist** observers who can see all free objects in space moving with uniform velocities. Equivalently, this is the same as saying that all free objects in space are moving at uniform velocities with respect to each other. ***And that is saying something about nature.***

An inertial observer himself or herself is (like) a free object. So we are really sitting on one free object and looking at other free objects. The first law defines the inertial observer. It defines who can observe the world "correctly". But there is more hidden inside this law. It says that this definition *is possible*. Let me define a new kind of observer - a *Gunertial* observer. You are a *Gunertial* observer if you find that ***all*** free objects are at rest. This sound like a nice definition, but you will be unable to find even one such *Gunertial* observer! I can always find one free object that is at rest with respect to me. But that doesn't help - because if I look at another free object, it probably would be moving relative to me. Once we find a *Gunertial* observer by looking at one free object, we cannot say this fellow will find all free objects at rest. And that is a serious problem. Newton on the other hand says that you **can definitely find "inertial" observers**. That if you find one inertial observer by looking at **one** free object - that fellow will do the job for **all** free objects. And not just this - all observers moving uniformly with respect to him/her will also do. In fact it says even more - only observers

moving uniformly with respect to him/her will do. So it is a very good law, with a definition and an existence statement hidden inside as well!

Summary: An inertial observer sees that the natural motion of objects is uniform. If she now sees the path of an object and finds that this object deviates from its natural path, she concludes there must be a force on it. She looks around for it and finds the object causing the force. The first object accelerates because of this force and that's why its motion deviates from its natural motion. The second law says that an object's deviation from natural motion is always caused by forces exerted by other physical objects (which therefore have to be around to influence the object). Additionally the second law also says that if you look at the rate of deviation in velocity - the acceleration - you will find simple patterns and laws for it.

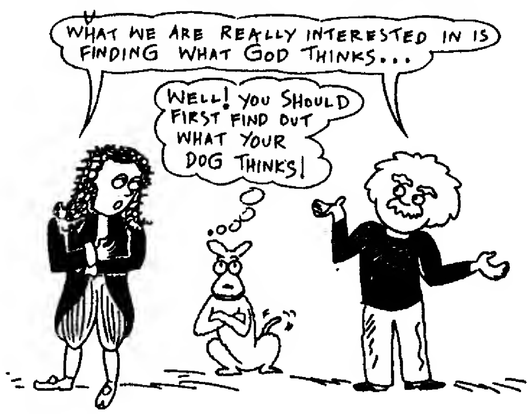
The first law tells us that uniform motion is the natural motion for an object. The second law tells us how much the object has deviated from this motion - due to the net force on it. The second law being a statement of deviation from natural motion does not contain the first law.

Conclusion

The idea of a free object that we have used so far is an approximation. One cannot really take an object too far away - because the Universe is full of objects, is expanding as a whole and decelerating because of gravity. Wherever we go, other objects will exert a force on our object. In our analysis above, we have assumed this effect to be negligible. But is it really?

In the late nineteenth century, Ernest Mach devoted a lot of time to this question. He argued that the mass of an object - its inertia - is just the gravitational pull on the object by the entire universe! Mass he suggested is not an independent property of an object but the measure of the entire universe's gravitational force on the object. He also argued that there is no absolute space and inertial frames are only relative to the average motion of the universe.

In the twentieth century, Einstein, Bohr, Heisenberg, Dirac, Feynman and others asked a lot of fundamental questions about space, time, force, mass, process of measurement and even about what knowledge means. Their ideas showed that Newtonian ideas of force and motion are good approximations when things are our size, but they don't work well for atoms and very small objects or for things moving very fast (close to the speed of light) or when objects are very massive (like near a star ten times the size of sun). But to understand most of the macroscopic things around us, we still use Newton's laws and forces. The programme of paying attention to forces that was started by Newton has been extremely fruitful and lot of our science and engineering owes its origins to this programme.

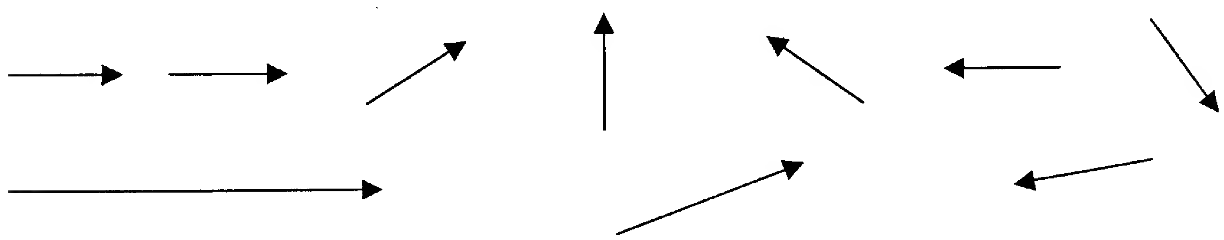


A Note on Vectors

Vector Basics

This is not a formal chapter on vectors. You must refer standards physics or mathematics books for that. This is just to help you get an intuitive feel for working with vectors and to be able to solve simple problems.

Mass is a number. We usually don't associate a direction with mass. Distance is a number - but we can associate a direction if we want to - "she went 5 m that way". A good way of talking about "distance + direction" is by drawing arrows. Below, we have seven arrows in the first row and 3 in the second row.



T: All the arrows in the first row are of equal length (let's say equal distance). Let's say an object moves from the tail to the head of the arrow. Is it doing the same thing in every arrow? In a sense - yes! All the arrows say the object is moving. But clearly the motions are not the same. In the second row the object moves more distance. But even in the first row, we know the motions are **not the same**. What is not the same about the motions?

S: They move in different directions.

T: What is different about the motion in the first arrows in the first and second rows?

S: The lengths are different.

T: What I want to know is the difference between the motions represented by the two arrows.

S: The second object moves more.

T: Can you give a numerical example ?

S: The first object moves 5 cm, the second object moves 15 cm. So they are different.

T: Very good. Now let's take the first and the fourth arrows. What is different about these motions?

S: Directions.

T: How will you quantify it with an example ?

S: The first object moves 5 cm to the right and the second object moves 5 cm upwards.

T: 5 cm is a very simple to understand. But this 'upwards' is tricky. What is upwards - out of the paper or 'north' of the paper? Anyway, let's move on arrows 1 and 3. What is different about these two motions?

S: The first object moves to the right 5 cm. And the second object moves 5 cms that way (showing).

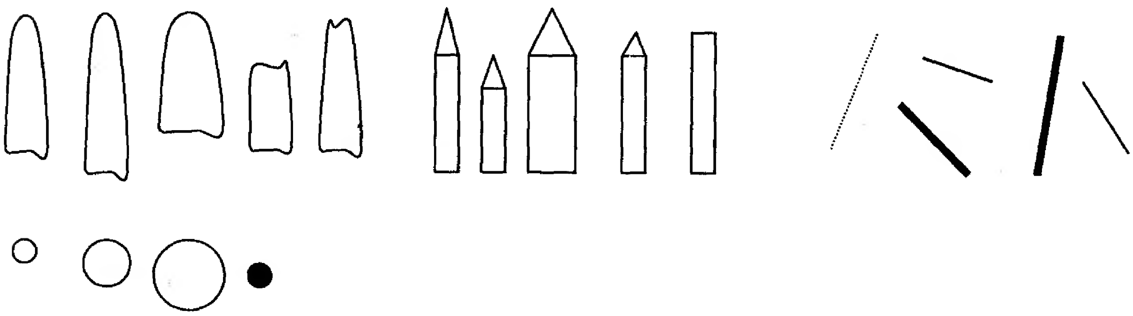
T: Exactly. The difference is easy to draw and show but tricky to quantify. If we are clever, we may say things like, it is at an angle of 30° to the horizontal, etc. But ultimately the point to recognize is this:

To differentiate distances, numbers will do (like 5 cm and 10 cm). But to differentiate motion, often we need direction, therefore we draw an arrow and say "it went like that".

Quantification is the key to science and life. It is one thing to say generally "he traveled a lot today". It is totally another to say "he traveled 23 km today". Numbers make our description concrete. Saying you have 5 apples is more concrete than saying you have some apples. But in describing different kinds of motion, we need to talk about the direction. We need to say "it went like that". We need to draw arrows. But drawing arrows doesn't seem very concrete.

People have been drawing arrows and saying "go like this and then like that, and then turn right" - possibly even when humans were living in caves. But only in recent times did people realize that these arrows are themselves good concrete mathematical description. This discovery converted these 'vague' arrows into concrete mathematical objects - **Vectors.**

We think of numbers as very natural - but they are really not so. If I say "Show me five", you will show me your fingers. But the question is where is the 5 in it? Nature only has objects. Let's say I place some chalks, some pencils and draw some lines on the table and then some circles.



We say there are 5 chalk pieces, 5 pencils and 5 lines and 4 circles. What does this mean? First thing is that you are saying you don't care about the differences in the chalk pieces - some are smaller, some are larger, some are broken or blunt. You decide not to worry about

these - they are all chalk pieces. They all have something in common, and which is something they don't share with a pencil - they are all chalks! You have constructed an abstract 'ideal' object called chalk piece. Similarly with 'pencils, 'lines'and 'circles'. You don't bother about thick and thin lines or dotted lines - they are all 'lines'. Now there is a bunch of chalk, bunch of pencils, a bunch of lines and a bunch of circle. Where is the five and four here?

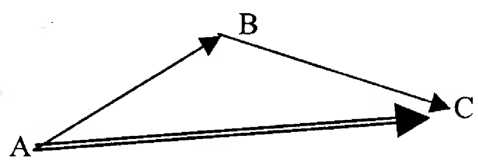
5 and 4 comes out of a mathematical process. You 'recognize' there is something common about the bunch of chalk, pencils and lines that is not present in bunch of the circles. With each chalk you can associate a pencil or a line. If you do this one after another, the chalk will end at the same time as the pencil and the line, but the circle will finish before. This one-to-one association (called mapping) is what creates numbers. By calling each association with names like 1,2,3,4,5,... we invent the concept of a number. Number is just the name for the commonality we discovered in these bunch of objects. By discovering this commonality, we abstracted out the concept of number from things that existed in nature.

One can think a lot more along these lines and it is very interesting. But instead, let's come back to our arrows. Numbers seem naturally concrete. Arrows don't. That's only because you learnt numbers in the first standard and are learning 'mathematical' arrows only now. You have had a lot of time to digest and use numbers.

Numbers are mathematical objects we humans created.
Vectors (Arrows) are also mathematical objects we created.

Number is a way of talking about "count of objects". Vector is way of talking about "the direction of motion". Just naming the count would not have made numbers concrete. We have a set of rules to manipulate numbers and play around with them. If we have 5 of something and 4 of the same thing, we can add and say we have 9 of something. There is an addition rule which says $5 + 4 = 9$. Rules like addition and subtraction is what makes numbers mathematical and useful. As long as people had arrows but no rules to play around with these arrows, it was not mathematical. Once people developed these rules, we invented the concept of the mathematical vector. Just like number rules, the vector rules are in principle simple.

Let's look at the addition rule. If you go from A to B and then B to C, you have effectively gone from A to C. This is shown in terms of arrows:



We say the arrow AB 'plus' arrow BC 'gives' arrow AC. Clearly this is a special kind of plus (addition) - this is not the addition we generally use with numbers. This is called arrow addition or more technically vector addition. The final arrow we get is called the resultant vector (we show it with a double line). When we add arrows, remember you cannot add the lengths - AB length + BC length is not equal to AC length. But the arrows add to give the final arrow.

The starting point for vectors is motion - how we move - distance and direction. We call this displacement. This is shown with an arrow. The idea is that the direction of the arrow is the

direction of the displacement and the length of the arrow is the length of the displacement. (Later we can say we will scale things up or down and have scale-drawings where 1 cm means 5 m etc. But the principle is the same.) **So a vector is an arrow with a specific direction and length.**

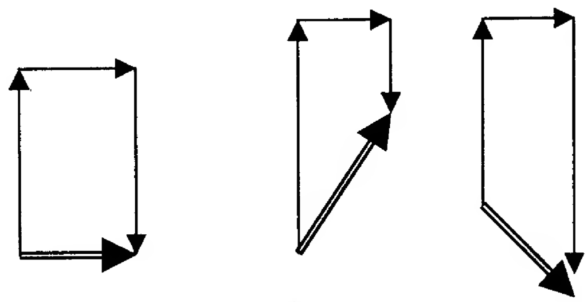
S: Arrow just shows the direction. Why do you need length and direction? We have numbers for length. Why can't we just have the direction alone for the arrows?

T: Good point. There are mathematical objects that only tell you direction - like angles. But for arrows that won't do. We need both. Let's see why. Suppose you go up and then come down, where will you be ?

S: At the same place.

T: Suppose you to go up, then go right and then go down. What have you effectively done?

S: You have gone right.



T: Let's draw arrows and see. The three diagrams on the right show three possibilities. The one you are talking about is the first diagram. When you usually talk of going up and coming down, you think you are going up and coming down the same distance. But this is not obvious. Even in the first question, you need not finally be at the same point - you could be above or below where you started. Clearly in the second and third figures on the right, the effective motion depends on how much you move up, how much you move down and how much you move right. It is therefore not enough if we only know the direction of motion. We also need to know the length. Only this can ensure that the arrow gives us enough information required to describe the motion.

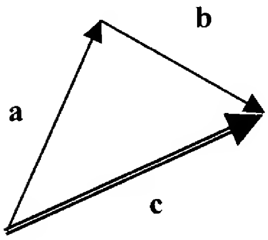
This is key to understanding vectors. People have been playing around with arrows for ages. Small children use arrows. People on the road often say "go like this and turn right and then turn left." They are using arrows. But they are non-mathematical arrows. Their arrows have only direction. They don't have length. Someone says "go straight, then turn at the first left, turn left again at the second left and once again turn at the first left". Try to draw a map. You can't figure out just from this information whether the final point must be below the starting point, above the starting point, to the right or to the left of the starting point. Effectively you know nothing about where your final destination is with respect to the initial point. Of course you may be able to reach the point - but in the process, you are actually measuring the lengths for the arrows. But the person giving you the information has not given the full information required to draw a map and locate the final position.

It is only arrows that have **length and direction**, that become mathematical - become vectors. Only when we have such special arrows (vectors), can we define addition of arrows as we did earlier. And it is this that makes these arrows mathematical. Without length, adding arrows

becomes impossible. Which is why lay-people can use arrows, but cannot quantify or add or do mathematics with them.

More on Vector Addition

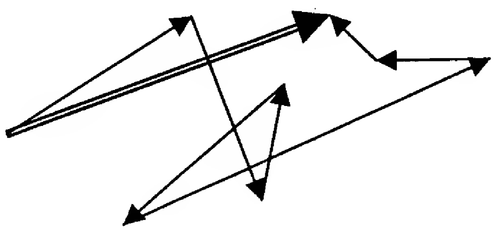
Arrow addition is a special kind of addition. Sometimes people think this is difficult. Usually the principle seems easy, but they are more worried about "how to add arrows". That's because they are thinking of adding without drawing a picture. We will learn how to do this - it is called adding components. But the point to recognize is that arrow adding is just what it says - add one arrow after another! The rest of the stuff is merely techniques to simplify the addition process - that does not make arrow addition a *difficult* idea. It is just a *different* idea. Let me explain this a bit more.



In the arrow addition diagram on the left, **a + b = c.** Many students are not comfortable with this. They feel somehow this is not quite the answer they want. That's because they are used to looking for numerical answers - answers which look at 5.4, 3.67, 8, 11/7, etc. Arrow as the answer to a question sounds somewhat incomplete. This is the cause for a lot of confusion and inability to use vectors effectively. When we use numbers as mathematical objects, we should expect answers in terms of numbers. But here our mathematical objects are arrows. We should learn to expect answers in terms of arrow. We should learn be comfortable with arrows as answers. **Arrow is a GOOD answer !**

An arrow is a different mathematical object from a number. You cannot therefore expect to work with arrows and get answers that look like numbers - you will get arrows. Algebra and arthimetic are the key tools in playing with numbers. Geometry (coordinate geometry, trigonometry and simple old geometry) is the tool which will help us play with arrows.

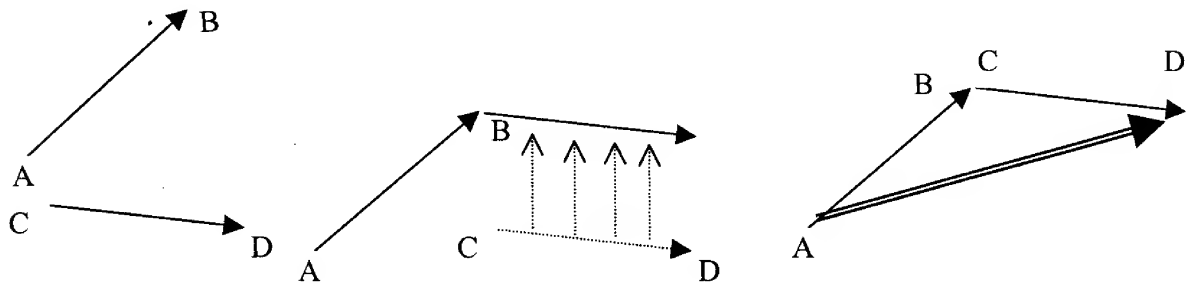
Unfortunately, we are so much more used to numbers than arrows, that often we want to use numbers to talk about arrows. This is doable (like using two numbers - length of arrow and angle). But it makes things a little bit difficult. Students exposed to writing arrow answers in terms of numbers therefore think that arrow addition is a 'difficult' concept. But the real difficulty is because you are trying to express a simple answer ('that arrow') as a combination of numbers. At least for thinking, one must learn to think of the entire arrow addition process in terms of arrows only - this makes it very easy and usable. Once you get used to this, you will see how easy arrow addition really is.



What if we have seven arrows to add? Simple. Just place them one after another and see where you end! See the diagram. We have seven arrows adding up. I have marked the final resultant arrow with a double line. If you follow the arrows, you see that all the arrows move in one direction, tail to head, tail to head, etc, but the resultant goes the other way. This is because the resultant

arrow is the equivalent of all the tail to head movements. The resultant is just saying that what these seven arrows have together achieved is what I am achieving in one go. So the resultant's starting point is the first arrow's starting point and the resultant's ending point is the last arrow's ending point.

What if the second arrow does not start from the head of the first arrow? Let's suppose I want to add the two arrows **AB** and **CD**. The usual way is to shift CD parallel to itself till C gets to B and then add the two arrows as we discussed earlier.



The diagrams above show this. So the resultant is shown in the third diagram by the double arrow.

S: But how can I shift C to B? Don't arrows start from a specific point?

T: This is an important point. If a person goes from A to B and another person goes from C to D. What do I mean by adding the two motions and what is the meaning of the effective motion? These are two totally different motions by two different persons and there is no meaning to adding these motions. On the other hand if I say the same person went from A to B and C to D and ask for the effective motion - again this has no meaning, because you can ask how she got from B to C!

S: Then why have you added these two arrows?

T: So far we have been talking about arrows showing the motion from one fixed point to another. Does an arrow have a starting point? We said mathematical arrows must have a length and direction - but never said anything so far about starting point. Do two arrows that have the same length, and same direction but different starting points (that means the arrows are parallel), say the same thing? This is a choice we have. Let's think about this. What does an arrow **AB** actually mean?

S: You move from A to B.

T: That's one interpretation. But you can also think of arrow **AB** as saying "your motion is the same as moving from A to B".

S: What's the difference?

T: Now you don't need to actually move from A to B. The arrow does not say where you start or end, it just says what your motion is - wherever you start. **AB** is just a way of talking

about your actual motion, without talking about where exactly this motion happened. If you are in Delhi and someone says go south x km and go west y km and you reach Bombay. Instead you start from Calcutta and similarly go south x km and go west y km and let's say you reach Chennai (this is not a fact, just a hypothetical example). Then you would be saying your motions are the same. But your starting and ending points are not.

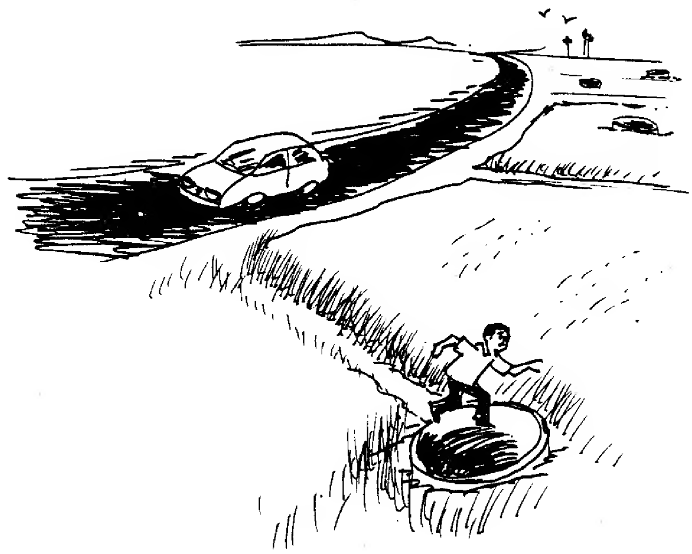
S: You mean to say arrows never specify the starting and ending points?

T: There are two kinds of arrows. The first kind of arrows talk only about motion - and not about starting points. They are called motion vectors. They you how you moved and how much you moved, but tell you nothing about your actual positions. The second kind of arrows talk about where you are. An example of the second kind of arrows are position vectors or movement vectors - which tell you what your position is from a fixed point or tell you that you have moved from X to Y. These vectors cannot be shifted around. They behave almost like the first kind, except for their immobility. When you say vector, we usually only mean the first kind of arrows - the ones that only talk about motion, without looking at the starting point. These are the ones more commonly used - and they contain no information about the starting point.

S: If we use them more, then how can we know where we are?

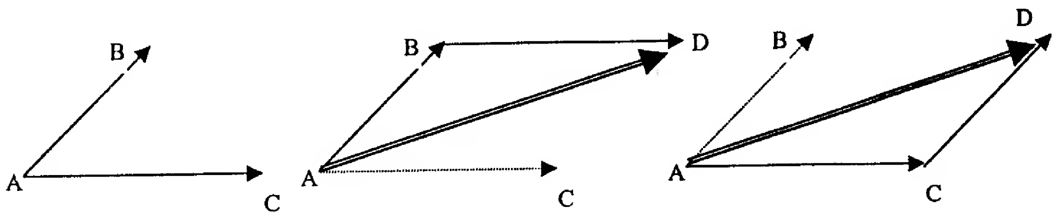
T: To find out where you are, you need to have one additional piece of information - starting point! But the point is we can do a lot even without knowing the starting point - that is the power of using these motion vectors instead of movement or position vectors. In most problems you only need to know how you are moving, you don't need to know where you are. Being more flexible and mobile, in such problems these vectors are very useful.

T: But there are problems where I need to know where I am. Let's say I am traveling on a long straight road with fields on both sides and let's say there are lots of wells dotting the fields. I decide to go off a road and walk 2 km perpendicular to the road inside the fields. My motion vector is always the same - an arrow perpendicular to the road and of 2 km length. But if I start at the wrong place, I will fall into the well! So in this problem the starting point is important - so either I use movement vectors or use motion vectors along with the knowledge of the starting point.

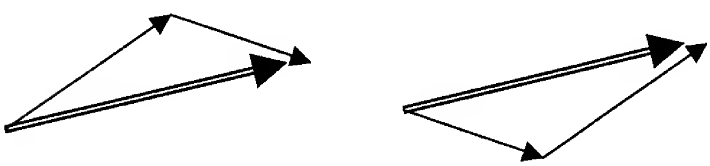


T: In the rest of our work, we assume that we deal with motion vectors - vectors that don't carry the starting and ending point information and only carry information about the motion. Then what does adding vector **AB** and vector **CD** mean? You are saying that you are starting somewhere at X (not necessarily at A) and executing a motion specified by arrow **AB** and then you are executing a motion specified by arrow **CD**. What is your net motion? (Note, we are not asking where are you at the end.) That's what **AB + CD** means. Since **CD** talks only about direction and length, you can shift **CD** parallel to itself and it is always the same vector. That's why we can shift a vector's tail to another vector's head and then add it. But it is important to understand that arrows have meanings assigned to them and when you add you must work within these meanings.

Which arrow's tail should I shift to which arrow's head? The question really is whether **AB + CD = CD + AB** ? And the answer is yes. Why? Let's assume that A and C are the same point. (This is not the same as saying B and C are the same point - then there is no shifting!)



I want to add the vectors **AC** and **AB** as shown in the first figure above. In the second figure I have shifted **AC** to **BD** - by moving it parallel. Therefore the answer (resultant) is **AD**. In the third figure instead of shifting **AC**, I have shifted **AB** to **CD** and added the two arrows and we still get the same **AD**. Since the two vectors **AD** and **AD** have the same direction and length, they are the same. (We don't know they start from the same A, because the two A's in the two figures are at different points and anyway we don't know where the starting point of **AB** is!) You can see that ABDC is a parallelogram and the answer is the diagonal vector. What if A and C are different points? Think and you will see the same logic holds! **It does not matter which vector's tail you move to which vector's head. The answer may be shifted a bit but it will be the same.**



In the figures on the left, I have added two vectors first by placing one vector's tail on the other vector's head and then by interchanging the two vectors. You can see that the resultant vector is the same, though the diagrams look different, bring the two diagrams together (align the resultants) and you will see the four vectors form a parallelogram. This knowledge is a useful one. **You can change the order in which you add vectors.**

The parallelogram law of addition of vectors: This is the same as what we did: **AC + AB = AD** in the earlier parallelogram diagram. We got it by shifting the arrows and adding them using the regular rule - head to tail. Some people make this into a vector addition rule and call it the parallelogram law of addition of vectors. *I feel this is this rule is unnecessary and non-intuitive. It replaces the simple geometric picture of addition of arrows with a non-intuitive parallelogram.* If you want to add two vectors starting at the same point (which is what the parallelogram law teaches you to do), you can always shift one to the head of the other and add using the regular arrow addition. The regular arrow addition we did is called the triangle law. It is really just adding arrows. *It is best to not use the parallelogram law and to just simply place one vector after another to find the resultant.*

Vector Subtraction: When we learnt addition we also learnt subtraction. Addition asks the question "If I add A and B, what do I get?". In subtraction, you know the answer and you want to know the question. "I got C, what should I add to A to get B ?" We say C-A is the answer. This is the principle - and is quite simple. Procedures for subtraction sometimes make it look difficult. But the principle is really simple. It is the same with vector subtraction. I know the final arrow, what should I add to the initial arrow to get this final arrow.

In the adjoining figure, I know the final answer is **AC**. What should I add to **AB** to get **AC**? See it this way: You want to get to C from A. But you have instead got to B. What should you do? Go from B to C. So the answer we want is the arrow from B to C! We write this as **AC - AB = BC.**

What if **C** and **A** are the same point? Then you went from A to B. But you really want to just stay at A. Just come back! Do a **BA**. **AC** now is just **AA** - this just means you did not move at all. **AA** is a zero vector. Let's call it **0**. So we get **AA - AB = 0 - AB = -AB = BA**. This is the definition of a negative vector. Negative vectors are even simpler than negative numbers - just go in the opposite direction! With this idea, we can say **AC - AB = AC + BA**. Vector addition is simplest when you write it such that the tail of the second vector is the same as the head of the first vector. Since we saw that you can add vectors in any order, let's make this a more convenient order. So instead of **AC + BA**, I will write it as **BA + AC**. But that makes it very easy. You went from **B** to **A**, then from **A** to **C**. So you went from **B** to **C**. The answer is **BC**! Some practice and thought will make this easy.

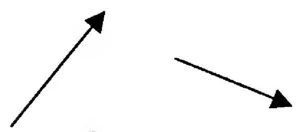
The best way to understand vectors is to draw a lot of arrows and ask a lot of questions and find answers for them, by moving them around. That is all there is to vectors. Shift arrows around. We will learn more techniques for adding vectors - using components. It is important to be conscious about what are the principles and what are the techniques. If something seems confusing, you can always go back to the first principles - adding arrows. This is a good strategy. But to be able to start from the basics, your basics must be strong. The only way this will happen is if you draw a lot of arrow diagrams and see what happens. Before you jump into the next section, you should practice vector addition and subtraction - without getting confused by the trigonometry and algebra that can make vectors look tough.

The power of vectors lies in its geometric and intuitive picture. Humans think well pictorially and vector helps in this. You must not lose this intuition at any cost. Every vector equation is a geometric statement about arrows - learn to see it this way. It takes some practice, but is very fruitful. When for example you work with force components, you will have equations for x-component, and y-component, etc. But really all that is happening in terms of arrows is that a number of force arrows add together to give you a single final force arrow and that determines the acceleration of the object. I am not saying you shouldn't work with components - that's why we are learning components. But once you get the answer, go back to the arrow picture to at least get a feel for what is really happening in terms of the arrows. Believe me - it will help a lot as you go along and will develop your understanding of mechanics and give you an intuitive feel that equations can never provide.

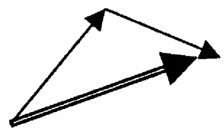
Vectors - Components

Is it easier to add 2 vectors or 7 vectors ?

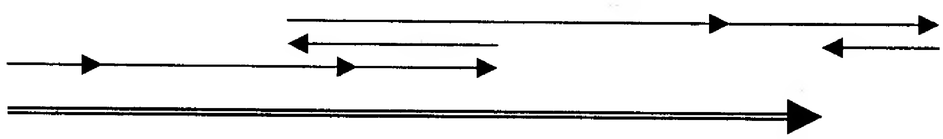
Some more playing around with leads to a very useful idea in vectors. Is it easier to add 2 vectors or 7 vectors? One would think adding two vectors is easier - but this is not always the case. Let's see why.



Let's say I have two vectors as shown on the left and I want to add them. To find the final vector, I have to put one vector's tail on the other vector's head and draw the final arrow as shown on the right. In principle this is simple, but it is also cumbersome. Suppose I tell you the two arrows in terms of lengths and angles, you will have to measure off lengths and angles using a protractor and draw lines etc. That's a lot of work - simple but tiring!



But what happens if there are 7 vectors that are all parallel and I want to add them? Do I still need to draw the vectors head to tail and find the final arrow and measure its length? We can of course do this - but the real question is whether we need to do this. Let's see. Try this problem - you have to add seven vectors. All are horizontal and are as follows: Right of length 2, Right of length 5, Right of length 3, Left of length 4, Right of length 7, Right of length 4 and Left of length 2.



In the above figure there are seven horizontal arrows as required in the problem - some moving to the right and some to the left. I have displaced a few arrows a bit above and below

just so that you can see what is adding to what. The first three arrows add up and then the fourth one goes back some distance, the fifth and the sixth arrows add more to the length and the seventh one reduces the length by a small amount. The final resultant arrow is the one with the double line. You already know its direction is horizontal. What is its final length? Simply $2 + 5 + 3 - 4 + 7 + 4 - 2 = 15$. So the resultant is 15 units long and towards the right horizontally. That's the answer! Unlike in the previous example, here the arrow lengths just add or subtract - there is really no need to draw the arrows. Let's arrows to the right are positive length and the to the left are negative lengths. Just add these lengths and you get the final arrow length. **NO NEED TO DRAW THE ARROWS!**

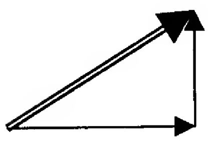
Why does this happen? Because the direction of all the arrows is the same (maybe positive or negative - but that's all). This same idea will work with vertical arrows or arrows in any direction - as long as all the arrows are in the same direction.

IMPORTANT LESSON

ADDING 7 ARROWS IN THE SAME DIRECTION IS A LOT EASIER THAN ADDING 2 ARROWS IN DIFFERENT DIRECTIONS. JUST ADD THE LENGTHS (WITH SIGN)!

S: But that's not fair - how do know that all the arrows will be parallel. More often the arrows will not be parallel. What do we do then?

T: True. But let's take this step by step. Now let's consider two arrows that are not parallel, but are rather perpendicular. **Is it easy adding two perpendicular arrows?**

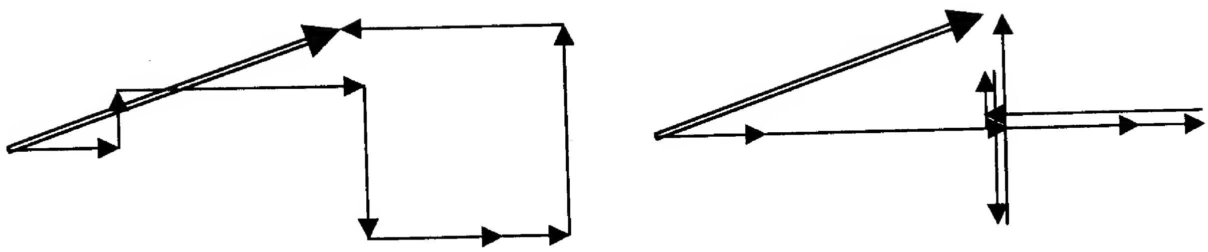


We can work with any two perpendicular arrows. But just to make it simpler, I am taking one of the arrows to be horizontal and the other to be vertical. Let's say the horizontal arrow has a length 3 units and the vertical one has a length 5 units. Then Pythogoras theorem tells us that the final arrow length must be $\sqrt{(9 + 25)} = \sqrt{34}$. Simple trigonometry also tells us that the resultant arrow is at an angle θ, such that $\tan \theta = 5/3$. So not only can we draw the arrow (which is easy for all sorts of arrows), but we can tell what length and angle the resultant arrow must have - without actually drawing it and measuring things.

So if you think about it a little, you see that adding perpendicular arrows (without drawing it out and measuring) is a relatively simple job. Of course not quite as simple as adding parallel arrows - but nevertheless simpler than adding random arrows.

What happens if there are several arrows - some of which are parallel and some of which are perpendicular?

Would you believe it? The two figures below are exactly the same!! Look carefully and you will see why. In the first figure, we have added a lot of parallel and perpendicular vectors one after the other and the final arrow seems reasonably difficult to find without actually drawing



the figure. In the second figure, we first added all the horizontal arrows - that's easy. Then we added all the vertical arrows - that's again easy. Then we just add the total horizontal arrow with the total vertical arrow - and that's also easy. So from the second figure we can easily say that the length of the final arrow must be square root of the sum of the squares of the sum of horizontal arrows and the sum of the vertical arrows. Spend sometime thinking about this. Put in numbers in the lengths and calculate the final arrow length both ways and see how this works.

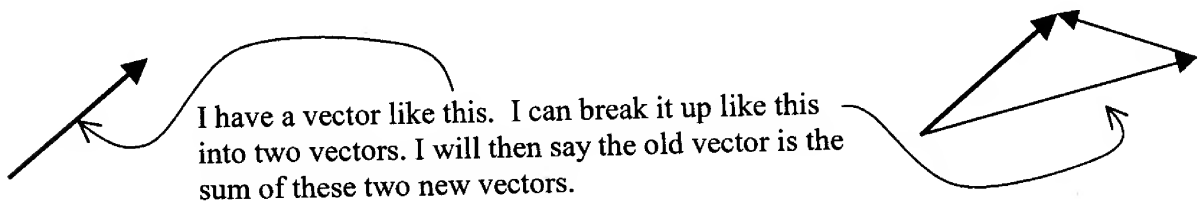
This is the crux of the component method. Add all the horizontals separately, add all the verticals separately and then add these together.

Note an interesting point in the figures. The vertical arrows contribute **nothing** to the horizontal motion and **the horizontal arrows don't disturb the vertical motion**. Horizontal and vertical arrows don't affect each other. This is another important idea we will keep using.

(*Exercise*: Do you need the vectors to be perpendicular for you to be able to do this? What if there are two sets of parallel vectors but which are not perpendicular to each other? We don't need this for our work here, but it is a good thing to think about. It leads to coordinate systems that are at an angle.)

S: This is all nice. But it only helps if we are given arrows that are all parallel or perpendicular. What if the arrows are at arbitrary angles?

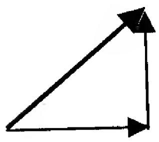
T: Then we will break them up! This process is called resolving or componentizing the vector.



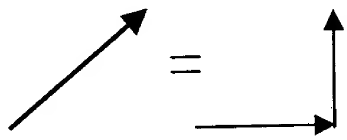
Of course you can break it up into many vectors, but two will suffice 2-dimensional problem and for 3-dimensional problems we will need to break it up into three. So although we can break up a vector into 5 or 6 parts, we seldom bother with it. Coming back to out 2 vectors - these two vectors together are the same as the old vector. So we can replace and work with these two vectors.

S: Why should anyone want to do that? This way you are increasing the number of vectors to keep in mind.

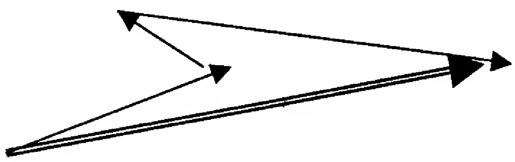
T: Yes. But I broke up (resolved) the vector into two arbitrary parts. Let's instead break it up into a horizontal part and a vertical part like as shown below.

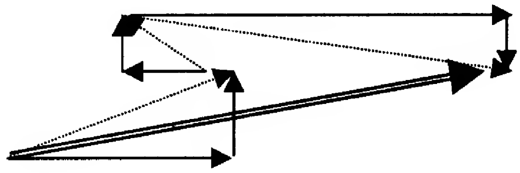


What we are saying really is that we can replace the earlier vector (at an angle) by two vectors which are horizontal and vertical. If we do this for all the vectors in a problem, then we just have horizontal and vertical vectors. An easy problem we already solved!

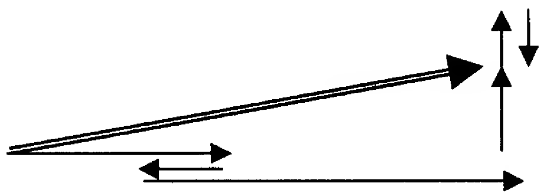


Let's try adding 3 vectors like this.





There are 3 figures here. The first is just adding 3 arrows - the simplest and clearest - at least geometrically. The second diagram breaks up each arrow into its x and y components. In the third figures the x-components are added together, the y are added together and the results of these two are added together.



The first figure has an advantage
It is visually simple.
The third figure has another advantage.
Helps you calculate easily.
Master and use both approaches.

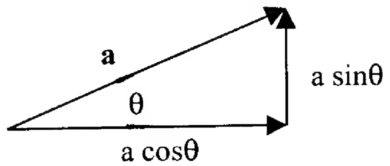
Note: You have more arrows in the last picture - but they are easy to add together. That's the point!

S: Having said that direct arrow diagrams are important, I can see you are trying to avoid drawing arrows and measuring the distance. But to find the components, don't we have to draw the arrow diagrams and measure the lengths?

T: You have hit the nail on the head. Mathematicians talk about calculating and measuring, but they don't like to actually take a scale and protractor and measure. All this jugglery is just so that we can avoid measuring arrow lengths with a scale! Coming to your question, they is

another way (without measuring to find the lengths of the components. We will use trigonometry.

T: Let's say we have a vector - with length a and making an angle θ with the horizontal. Then the horizontal component is a $\cos\theta$ and the vertical component is a $\sin\theta$. You can show this with simple trigonometry.



The horizontal component of **a** is the 'projection of **a** vector' along the x-direction = a $\cos\theta$ and the vertical component of **a** is the 'projection of **a** vector' along the y-direction = a $\sin(90^o-\theta)$ = a $\sin\theta$.

So putting everything together, we have the following rule for adding vectors using components!

1. Break up all the vectors into x and y directions - using their projections.
2. **Instead of** adding the vectors, add their components.
3. Add all the x-components together. This is simple - just add the lengths (positive and negative). Add all the y-components together in the same way.
4. Now you have two perpendicular vectors - add them using Pythagoras theorem and you can find the angle using $\tan\theta = y/x$.

This seems tedious - but often is very routine and simple. The only problem with any procedure like this is that it takes you away from a simple geometric picture of what happens. So make sure you also constantly think in direct vector terms even if you are using components. It will really help you develop a feel for vectors, forces and for physical situations.

Many students resolve and then still retain the original vector - this leads to double counting. You can either keep the original vector or keep the two components. **You can't have both.** And sometimes students resolve a vector and re-resolve one of the components into another direction. This usually lands them in trouble. When re-resolving, they should make sure the re-resolve both the components. That way they get 4 vectors in the second round of resolving which together gives them the old vector. Usually it is best practice to never re-resolve a vector. Find the directions you want to resolve in and then resolve the vector into its components.

How to write a vector?

Usually people want vectors in terms of angles and lengths. But now we know that every vector (on the page) can be written in terms of two vectors - one horizontal and another vertical. So instead of writing a vector in terms of length and angle we can always write it in terms of components. This is actually the best way to write a vector - as sum of two (or three for 3-D problems) vectors that you already know. For this we use unit vectors in the x and y direction. Call them **i** and **j** vectors. This means **i** points towards the right and has length 1 unit and **j** similarly points in the upwards direction has a unit length. 2**i** or 3**j** would mean a vector twice or thrice the length in the x-direction or the y-direction respectively.

S: How can we use this to write any other vector?

T: Remember the earlier vector **a** that had a horizontal component of $a\cos\theta$ and a vertical component of $a\sin\theta$. This vector is basically the sum of $a\cos\theta\,\mathbf{i}$ and $a\sin\theta\,\mathbf{j}$. So:

$$\mathbf{a} = a\cos\theta\,\mathbf{i} + a\sin\theta\,\mathbf{j}$$

S: But what is the sum ?

T: The sum is the sum! **That is the answer**! We have simplified it to the best possible extent. We have to learn that answers can be in many forms. They need not be in numbers always. For example in complex numbers we similarly write answers in terms of a 'plus'. Similarly here, the vector is the sum. All you are saying is "go $a\cos\theta$ to the right and $a\sin\theta$ up and that is the arrow". Earlier I asked you to become comfortable with arrows as answers. Now you just have to learn to be comfortable with a sum of two arrows (components) as the answer.

S: You mean all vectors will be given like $3\mathbf{i} + 4\mathbf{j}$, or $6\mathbf{i} + 7.3\mathbf{j}$, etc ? And we can leave the answer in terms of the sum?

T: Yes.

S: But that will make adding them very simple.

T: Exactly. You just have to add up two sets of numbers and write the final answer! If someone wants the final length, just use Pythagoras theorem and to find angle use trigonometry.

S: But there must be a catch somewhere...

T: Well... In Physics, you often only know the length and angle of some of the vectors. So you have to write each vector in terms of components and then add them. But otherwise there really is no catch. Vectors are very easy. Just get used to them. The best way to work with vectors is to keep two pictures in your head - the direct vector picture for visualization and the component picture for calculations.

Independence of Horizontal and Vertical Directions

An important point we just touched upon earlier (and which you will find popping up again and again) is the idea of horizontal and vertical (more generally perpendicular directions) being independent of each other. What does this mean? It means all the following:

1. If you resolve a horizontal vector in the vertical direction, you get zero! (Check in the formula).
2. If you resolve a vertical vector in the horizontal direction, you get zero!
3. Vertical motion/movement is not affected by horizontal motion/movement. Horizontal motion is not affected by vertical motion.
4. The horizontal distance moved by some random vector is its horizontal component. The vertical component plays no role in this. Vice-versa with the vertical component.
5. Horizontal motion is independent of vertical motion and vice-versa.
6. If two vectors are equal then each of their components are equal.

Think about each statement. All of the above statements are the same thing. If you draw a lot of vector diagrams and look at the horizontal distances moved and vertical distances moved - and see what makes the point move up or right, you will see the meaning of this independence.

Force is a Vector !

That's all I need to really say and you can work out all the properties of force from this statement. But to spare you the trouble, we will look at some implications here.

Force has direction. It is not just a number. This you already know. It matters what direction you push in. But that's not all. When you say it is a vector, you are saying it behaves like a mathematical arrow (like displacement) - it adds like an arrow. And force is an arrow with length and direction - both. (And force arrows can be shifted parallel to themselves for addition. For some purposes, they can't be shifted but we won't bother about that here.)

'Force is a vector' also means that it can be written in terms of components and as sum of two perpendicular vectors - as $F_x\,\mathbf{i} + F_y\,\mathbf{j}$. This statement says that we should work with force the same way we did with earlier motion arrows. You can't say a force of 5 Newtons and 6 Newtons gives a total force of 11 Newtons. You have to draw arrows and find the length of the arrow (or equivalently write the components and add them.)

S: **Why is force a vector?** Why is it not simply a number?

T: Interesting question. But no one knows why! We only know it is a vector. Nature seems to make things difficult for you! Since we are trying to describe nature and vectors seem the simplest way to describe her, we accept that Force is a vector and work with it. That does not mean we know why Force is a vector. It is just that 'the world is like that'. We can only see if it is consistent with other laws that we know. You know force causes acceleration. Acceleration has a direction and behaves just like motion arrows (vectors). Force is a vector that causes acceleration-a vector in the same direction as the force. This is a consistent picture.

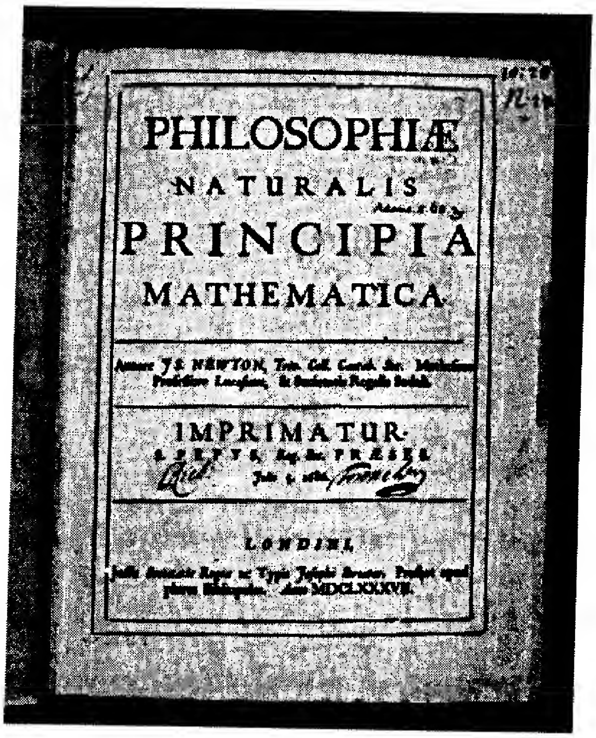
$\mathbf{F} = m\mathbf{a}$ is really a vector statement. **F** has a direction and **a** has the same direction. Using the component independence idea, one can say the horizontal component of **F** causes the horizontal component of **a**, vertical component of **F** causes the vertical component of **a**. If there are several forces acting on a body, you take the horizontal components of all the forces - they and only they together cause the horizontal acceleration. The vertical force components don't affect the horizontal acceleration. Similarly, the horizontal components don't affect the vertical acceleration. This is a very important statement about forces. And the Vector Picture naturally incorporates this into its structure.

And here are some more addition rules for forces.

1. Several Forces on the same body can be added as vectors - place tail to head and add!
2. This resultant force is what causes the acceleration of the object. We often resolve acceleration into several vectors. But there is only one acceleration for an object. On the other hand there are often several forces on an object and we are finding the net force. The **net** force gives **the** acceleration. There is no such thing as the net acceleration.
3. You can resolve forces into components and add these components to find the net force.
4. You cannot add forces on different objects - this addition has no meaning.
5. Forces that are perpendicular are independent. This means the accelerations they cause are independent. So if we have several forces, we can resolve them into horizontal and vertical components and say that horizontal forces cause horizontal acceleration and the vertical force components cause the vertical acceleration.

Appendix

Isaac Newton's Principia



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

IMPRIMATUR

LONDINI,

THE
MATHEMATICAL
PRINCIPLES
OF
Natural Philosophy.

By Sir *ISAAC NEWTON.*
Translated into *English* by ANDREW MOTTE.
July 5, 1686

Below is a selected extract from Newton's Original Principia. The language and spelling is Old English with long sentences and sometimes hard to read. But it is always a thrill to read the original. The clarity with which Newton looks at the concepts of force, action, reaction, space, time, etc is really amazing considering he wrote this in 1686!

Definitions

DEFINITION I: The Quantity of Matter is the measure of the same, arising from its density and bulk conjunctly.

Thus air of a double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction; and of all bodies that are by any causes whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter every where under the name of Body or Mass. And the same is known by the weight of each body: For it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shewn hereafter.

DEFINITION II: The Quantity of Motion is the measure of the same, arising from the velocity and quantity of matter conjunctly.

The motion of the whole is the Sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.

DEFINITION III: ***The* Vis Insita*, or Innate Force of Matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.***

This force is ever proportional to the body whose force it is; and differs nothing from the inactivity of the Mass, but in our manner of conceiving it. A body from the inactivity of matter, is not without difficulty put out of its state of rest or motion. Upon which account, this *Vis insita*, may, by a most significant name, be called *Vis Inertiæ*, or Force of Inactivity. But a body exerts this force only, when another force impress'd upon it, endeavours to change its condition; and the exercise of this force may be considered both as resistance and impulse: It is resistance in so far as the body, for maintaining its present state withstands the force impressed; it is impulse, in so far as the body, by not easily giving way to the impress'd force of another, endeavours to change the state of that other. Resistance is usually ascrib'd to bodies at rest, and impulse to those in motion: But motion and rest, as commonly conceived, are only relatively distinguished; nor are those bodies always truly at rest, which commonly are taken to be so.

DEFINITION IV: An impress'd force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

This force consists in the action only; and remains no longer in the body, when the action is over. For a body maintains every new state it acquires, by its *Vis Inertiæ* only. Impress'd forces are of different origines; as from percussion, from pressure, from centripetal force.

DEFINITION V: A Centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

Of this sort is Gravity by which bodies tend to the centre of the Earth; Magnetism, by which iron tends to the loadstone; and that force, whatever it is, by which the Planets are perpetually drawn aside from the rectilinear motions, which otherwise they wou'd pursue, and made to revolve in curvilinear orbits. A

stone, whirled about in a sling, endeavours to recede from the hand that turns it; and by that endeavour, distends the sling, and that with so much the greater force, as it is revolv'd with the greater velocity; and as soon as ever it is let go, flies away. That force which opposes it self to this endeavour, and by which the sling perpetually draws back the stone towards the hand, and retains it in its orbit, because 'tis directed to the hand as the centre of the orbit, I call the Centripetal force. And the same thing is to be understood of all bodies, revolv'd in any orbits. They all endeavour to recede from the centres of their orbits; and were it not for the opposition of a contrary force which restrains them to, and detains them in their orbits, which I therefore call Centripetal, would fly off in right lines, with an uniform motion. A projectile, if it was not for the force of gravity, would not deviate towards the Earth, but would go off from it in a right line and that with an uniform motion, if the resistance of the Air was taken away. 'Tis by its gravity that it is drawn aside perpetually from its rectilinear course, and made to deviate towards the Earth, more or less, according to the force of its gravity, and the velocity of its motion. The less its gravity is, for the quantity of its matter, or the greater the velocity with which it is projected, the less will it deviate from a rectilinear course, and the farther it will go. If a leaden ball projected from the top of a mountain by the force of gun-powder with a given velocity, and in a direction parallel to the horizon, is carried in a curve line to the distance of two miles before it falls to the ground; the same, if the resistance of the Air was took away, with a double or decuple velocity, would fly twice or ten times as far. And by increasing the velocity, we may pleasure increase the distance to which it might be projected, and diminish the curvature of the line, which it might describe, till at last it should fall at the distance of 10, 30, or 90 degrees, or even might go quite round the whole Earth before it falls; or lastly, so that it might never fall to the Earth, but go forwards into the Celestial Spaces, and proceed in its motion *in infinitum*. And after the same manner, that a projectile, by the force of gravity, may be made to revolve in an orbit, and go round the whole Earth, the Moon also, either by the force of gravity, if it is endued with gravity, or by any other force, that impells it towards the Earth, may be perpetually drawn aside towards the Earth, out of the rectilinear way, which by its innate force it would pursue; and be made to revolve in the orbit which it now describes: nor could the Moon without some such force, be retain'd in its orbit. If this force was too small, it would not sufficiently turn the Moon out of a rectilinear course: if it was too great, it would turn it too much, and draw down the Moon from its orbit towards the Earth. It is necessary, that the force be of a just quantity, and it belongs to the Mathematicians to find the force, that may serve exactly to retain a body in a given orbit, with a given velocity; and *vice versa*, to determine the curvilinear way, into which a body projected from a given place, with a given velocity, may be made to deviate from its natural rectilinear way, by means of a given force. The quantity of any Centripetal Force may be considered as of three kinds, Absolute, Accelerative, and Motive.

DEFINITION VI: The absolute quantity of a centripetal force is the measure of the same, proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about.

Thus the magnetic force is greater in one load-stone and less in another, according to their sizes and strength.

DEFINITION VII: The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.

Thus the force of the same load-stone is greater at a less distance, and less at a greater: also the force of gravity is greater in valleys, less on tops of exceeding high mountains; and yet less (as shall be hereafter shewn) at greater distances from the body of the Earth; but at equal distances, it is the same every where; because (taking away, or allowing for, the resistance of the Air) it equally accelerates all falling bodies, whether heavy or light, great or small.

DEFINITION VIII: The motive quantity of a centripetal force, is the measure of the same, proportional to the motion which it generates in a given time.

Thus the Weight is greater in a greater body, less in a less body; it is greater near to the Earth, and less at remoter distances. This sort of quantity is the centripetency, or propension of the whole body towards the centre, or as I may say, its Weight; and it is ever known by the quantity of a force equal and contrary to it, that is just sufficient to hinder the descent of the body. These quantities of Forces, we may for brevity's sake call by the names of Motive, Accelerative and Absolute forces; and for distinction sake consider them, with respect to the Bodies that tend to the centre; to the Places of those bodies; and to the Centre of force towards which they tend: That is to say, I refer the Motive force to the Body, as an endeavour and propensity of the whole towards a centre, arising from the propensities of the several parts taken together; the Accelerative force to the Place of the body, as a certain power or energy diffused from the centre to all places around to move the bodies that are in them; and the Absolute force to the Centre, as indued with some cause, without which those motive forces would not be propagated through the spaces round about; whether that cause is some central body, (such as is the Load-stone, in the centre of the force of Magnetism, or the Earth in the centre of the gravitating force) or any thing else that does not yet appear. For I here design only to give a Mathematical notion of those forces, without considering their Physical causes and seats. Wherefore the Accelerative force will stand in the same relation to the Motive, as celerity does to motion. For the quantity of motion arises from the celerity, drawn into the quantity of matter; and the motive force arises from the accelerative force drawn into the same quantity of matter. For the sum of the actions of the Accelerative force, upon the several particles of the body, is the Motive force of the whole. Hence it is, that near the surface of the Earth, where the accelerative gravity, or force productive of gravity in all bodies is the same, the motive gravity or the Weight is as the Body: but if we should ascend to higher regions, where the accelerative gravity is less, the Weight would be likewise diminished, and would always be as the product of the Body, by the Accelerative gravity. So in those regions, where the accelerative gravity is dimished into one half, the Weight of a body two or three times less, will be four or six times less.

I likewise call Attractions and Impulses, in the same sense, Accelerative, and Motive; and use the words Attraction, Impulse or Propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not Physically but Mathematically: Wherefore, the reader is not to imagine, that by those words, I any where take upon me to define the kind, or the manner of any Action, the causes or the physical reason thereof, or that I attribute Forces, in a true and Physical sense, to certain centres (which are only Mathematical points); when at any time I happen to speak of centres as attracting or as endued with attractive powers.

SCHOLIUM.

Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define Time, Space, Place and Motion, as being well known to all. Only I must observe, that the vulgar conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which, it will be convenient to distinguish them into Absolute and Relative, True and Apparent, Mathematical and Common.

I. **Absolute, True, and Mathematical Time**, of it self, and from its own nature flows equably without regard to any thing external, and by another name is called Duration: Relative, Apparent, and Common Time is some sensible and external (whether accurate or unequable) measure of Duration by the means of motion, which is commonly used instead of True time; such as an Hour, a Day, a Month, a Year.

II. **Absolute Space**, in its own nature, without regard to any thing external, remains always similar and immoveable. Relative Space is some moveable dimension or measure of the absolute spaces; which our senses determine, by its position to bodies; and which is vulgarly taken for immoveable space; Such is the dimension of a subterraneous, an aereal, or celestial space, determin'd by its position in respect of the Earth. Absolute and Relative space, are the same in figure and magnitude; but they do not remain always numerically the same. For if the Earth, for instance, moves; a space of our Air, which relatively and in respect of the Earth, remains always the same, will at one time be one part of the absolute space into which the Air passes; at another time it will be another part of the same, and so, absolutely understood, it will be perpetually mutable.

III. **Place** is a part of space which a body takes up, and, is according to the space, either absolute or relative. I say, a Part of Space; not the situation, nor the external surface of the body. For the places of equal Solids, are always equal; but their superficies, by reason of their dissimilar figures, are often unequal. Positions properly have no quantity, nor are they so much the places themselves, as the properties of places. The motion of the whole is the same thing with the sum of the motions of the parts, that is, the translation of the whole, out of its place, is the same thing with the sum of the translations of the parts out of their places; and therefore the Place of the whole, is the same thing with the Sum of the places of the parts, and for that reason, it is internal, and in the whole body.

IV. **Absolute motion**, is the translation of a body from one absolute place into another; and Relative motion, the translation from one relative place into another. Thus in a Ship under sail, the relative place of a body is that part of the Ship, which the Body possesses; or that part of its cavity which the body fills, and which therefore moves together with the Ship: And Relative rest, is the continuance of the Body in the same part of the Ship, or of its cavity. But Real, absolute rest, is the continuance of the Body in the same part of that Immoveable Space, in which the Ship it self, its cavity, and all that it contains, is moved. Wherefore, if the Earth is really at rest, the Body, which relatively rests in the Ship, will really and absolutely move with the same velocity which the Ship has on the Earth. But if the Earth also moves, the true and absolute motion of the body will arise, partly from the true motion of the Earth, in immoveable space; partly from the relative motion of the Ship on the Earth: and if the body moves also relatively in the Ship; its true motion will arise, partly from the true motion of the Earth, in immoveable space, and partly from the relative motions as well of the Ship on the Earth, as of the Body in the Ship; and from these relative motions, will arise the relative motion of the Body on the Earth. As if that part of the Earth where the Ship is, was truly mov'd toward the East, with a velocity of 10010 parts; while the Ship it self with a fresh gale, and full sails, is carry'd towards the West, with a velocity express'd by 10 of those parts; but a Sailor walks in the Ship towards the East, with 1 part of the said velocity: then the Sailor will be moved truly and absolutely in immoveable space towards the East with a velocity of 10001 parts, and relatively on the Earth towards the West, with a velocity of 9 of those parts.

Absolute time, in Astronomy, is distinguish'd from Relative, by the Equation or correction of the vulgar time. For the natural days are truly unequal, though they are commonly consider'd as equal, and used for a measure of time: Astronomers correct this inequality for their more accurate deducing of the celestial motions. It may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the True, or equable progress, of Absolute time is liable to no change. The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore it ought to be distinguish'd from what are only sensible measures thereof; and out of which we collect it, by means of the Astronomical equation. The necessity of which Equation, for determining the Times of a phænomenon, is evinc'd as well from the experiments of the pendulum clock, as by eclipses of the Satellites of Jupiter. As the order of the parts of Time is immutable, so also is the order of the parts of Space. Suppose those parts to be

mov'd out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the Places as well of themselves as of all other things. All things are placed in Time as to order of Succession; and in Space as to order of Situation. It is from their essence or nature that they are Places; and that the primary places of things should be moveable, is absurd. These are therefore the absolute places; and translations out of those places, are the only Absolute Motions.

But because the parts of Space cannot be seen, or distinguished from one another by our Senses, therefore in their stead we use sensible measures of them. For from the positions and distances of things from any body consider'd as immoveable, we define all places: and then with respect to such places, we estimate all motions, considering bodies as transfer'd from some of those places into others. And so instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs: but in Philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them.

For it may be that there is no body really at rest, to which the places and motions of others may be referr'd. But we may distinguish Rest and Motion, absolute and relative, one from the other by their Properties, Causes and Effects. It is a property of Rest, that bodies really at rest do rest in respect of one another. And therefore as it is possible, that in the remote regions of the fixed Stars, or perhaps far beyond them, there may be some body absolutely at rest; but impossible to know from the position of bodies to one another in our regions, whether any of these do keep the same position to that remote body; it follows that absolute rest cannot be determined from the position of bodies in our regions.

It is a property of motion, that the parts, which retain given positions to their wholes, do partake of the motions of those wholes. For all the parts of revolving bodies endeavour to recede from the axe of motion; and the impetus of bodies moving for-wards, arises from the joint impetus of all the parts. Therefore, if surrounding bodies are mo' 'd, those that are relatively at rest within them, will partake of their motion. Upon which account, the true and absolute motion of a body cannot be determin'd by the translation of it from those which only seem to rest: For the external bodies ought not only to appear at rest, but to be really at rest. For otherwise, all included bodies, beside their translation from near the surrounding ones, partake likewise of their true motions; and tho' that translation was not made they would not be really at rest, but only seem to be so. For the surrounding bodies stand in the like relation to the surrounded, as the exterious part of a whole does to the interiour, or as the shell does to the kernel; but, if the shell moves, the kernel will also move, as being part of the whole, without any removal from near the shell.

A property near a kin to the preceding, is this, that if a place is mov'd, whatever is placed therein moves along with it; and therefore a body, which is mov'd from a place in motion, partakes also of the motion of its place. Upon which account all motions from places in motion, are no other than parts of entire and absolute motions; and every entire motion is composed out of the motion of the body out of its first place, and the motion of this place out of its place, and so on; until we come to some immoveable place, as in the before mention'd example of the Sailor. Wherefore entire and absolute motions can be no otherwise determin'd than by immoveable places; and for that reason I did before refer those absolute motions to immoveable places, but relative ones to moveable places. Now no other places are immoveable, but those that, from infinity to infinity, do all retain the same given positions one to another; and upon this account, must ever remain unmov'd; and do thereby constitute, what I call, immoveable space. The Causes by which true and relative motions are distinguished, one from the other, are the forces impress'd upon bodies to generate motion. True motion is neither generated nor alter'd, but by some force impress'd upon the body moved: but relative motion may be generated or alter'd without any force impress'd upon the body. For it is sufficient only to impress some force on other bodies with which the former is

compar'd, that by their giving way, that relation may be chang'd, in which the relative rest or motion of this other body did consist. Again, True motion suffers always some change from any force impress'd upon the moving body; but Relative motion does not necessarily undergo any change, by such forces. For if the same forces are likewise impress'd on those other bodies, with which the comparison is made, that the relative position may be preserved, then that condition will be preserv'd, in which the relative motion consists. And therefore, any relative motion may be changed, when the true motion remains unalter'd, and the relative may be preserv'd, when the true suffers some change. Upon which accounts, true motion does by no means consist in such relations.

The Effects which distinguish absolute from relative motion are, the forces of re-ceding from the axe of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion. If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then fill'd with water, and held at rest together with the water; after by the sudden action of another force, it is whirl'd about the contrary way, and while the cord is untwisting it self, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move: but the vessel, by gradually comunicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming it self into a concave figure, (as I have experienced) and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shews its endeavour to recede from the zes of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, discovers it self, and may be measured by this endeavour. At first, when the relative motion of the water in the vessel was greatest it produc'd no endeavour to recede from the axe: the water shew'd no tendency to the circumference, nor any ascent towards the sides of the vessel, but remain'd of a plain surface, and therefore its True circular motion had not yet begun. But afterwards, when the relative motion of the water had decreas'd, the ascent thereof towards the sides of the vessel, prov'd its endeavour to recede from the axe; and this endeavour shew'd the real circular motion of the water perpetually increasing, till it had acquir'd its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavour does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defin'd by such translations. There is only one real circular motion of any one revolving body, corresponding to only one power of endeavouring to recede from its axe of motion, as its proper and adequate effect: but relative motions in one and the same body are innumerable, according to the various relations it bears to external bodies, and like other relations, are altogether destitute of any real effect, any otherwise than they may perhaps participate of that one only true motion. And therefore in their system who suppose that our heavens, revolving below the sphere of the fixt Stars, carry the Planets along with them; the several parts of those heavens, and the Planets, which are indeed relatively at rest in their heavens, do yet really move. For they change their position one to another (which never happens to bodies truly at rest) and being carried together with their heavens, participate of their motions, and as parts of revolving wholes, endeavour to recede from the axe of their motions. Wherefore relative quantities, are not the quantities themselves, whose names they bear, but those sensible measures of them (either accurate or inaccurate) which are commonly used instead of the measur'd quantities themselves. And if the meaning of words is to be determin'd by their use; then by the names Time, Space, Place and Motion, their measures are properly to be understood; and the expression will be un-usual, and purely Mathematical, if the measured quantities themselves are meant. Upon which account, they do strain the Sacred Writings, who there interpret those words for the measur'd quantities. Nor do those less defile the purity of Mathematical and Philosophical truths, who confound real quantities themselves with their relations and vulgar measures.

It is indeed a matter of great difficulty to discover, and effectually to distinguish, the True motions of particular bodies from the Apparent: because the parts of that immoveable space in which those motions are perform'd, do by no means come under the observation of our senses. Yet the thing is not altogether desperate; for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions; partly from the forces, which are the causes and effects of the true motions. For instance, if two globes kept at a given distance one from the other, by means of a cord that connects them, were revolv'd about their common centre of gravity; we might, from the tension of the cord, discover the endeavour of the globes to recede from the axe of their motion, and from thence we might compute the quantity of their circular motions. And then if any equal forces should be impress'd at once on the alternate faces of the globes to augment or diminish their circular motions; from the encrease or decrease of the tension of the cord, we might infer the increment or decrement of their motions; and thence would be found, on what faces those forces ought to be impress'd, that the motions of the golbes might be most augmented, that is, we might discover their hindermost faces, or those which, in the circular motion, do follow. But the faces which follow being known, and consequently, the opposite ones that precede, we should likewise know the determination of their motions. And thus we might find both the quantity and the determination of this circular motion, ev'n in an immense vaccum, where there was nothing external or sensible with which the globes could be compar'd. But now if in that space some remote bodies were plac'd that kept always a given position one to another, as the Fixt Stars do in our regions; we cou'd not indeed determine from the relative translation of the globes among those bodies, whether the motion did belong to the globes or to the bodies. But if we observ'd the cord, and found that its tension was that very tension which the motions of the golbes requir'd, we might conclude the motion to be in the globes, and the bodies to be at rest; and then, lastly, from the translation of the globes among the bodies, we should find the determination of their motions. But how we are to collect the true motions from their causes, effects, and apparent differences; and *vice versa*, how from the motions, either true or apparent, we may come to the knowledge of their causes and effects, shall be explain'd more at large in the following Tract. For to this end it was that I compos'd it.

Axioms or Laws of Motion

LAW I. Every body persevers in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impress'd thereon.

Projectiles persevere in their motions, so far as they are not retarded by the resistance of the air, or impell'd downwards by the force of gravity. A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the Planets and Comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.

LAW II. The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impress'd altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force) if the body moved before, is added to or subducted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joyned, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III. To every Action there is always opposed an equal Reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tyed to a rope, the horse (if I may so say) will be equally drawn back towards the stone: For the distended rope, by the same endeavour to relax or unbend it self, will draw the horse as much towards the stone, as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other; that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities, but in the motions of bodies; that is to say, if the bodies are not hinder'd by any other impediments. For because the motions are equally changed, the changes of the velocities made towards contrary parts, are reciprocally proportional to the bodies. This Law takes place also in Attractions, as will be proved in the next Scholium.

COROLLARY I. A body by two forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides, by those forces apart. (Pl. 1. Fig. 1.)

If a body in a given time, by the force M impress'd apart in the place A, should with an uniform motion be carried from A to B; and by the force N impress'd apart in the same place, should be caried from A to C: compleat the parallelogram ABCD, and by both forces acting together, it will in the same time be carried in the diagonal from A to D. For since the force N acts in the direction of the line AC, parallel to BD, this force (by the second law) will not at all alter the velocity generated by the other force M, by which the body is carried towards the line BD. The body therefore will arrive at the line BD in the same time, whether the force N be impress'd or not; and therefore at the end of that time, it will be found somewhere in the line BD. By the same argument, at the end of the same time it will be found somewhere in the line CD. Therefore it will be found in the point D, where both lines meet. But it will move in a right line from A to D by Law I.

COROLLARY II. And hence is explained the composition of any one direct force AD, out of any two oblique forces AB and BD; and, on the contrary the resolution of any one direct force AD into two oblique forces AB and BD: which composition and resolution are abundantly confirmed from Mechanics. (Fig. 2.)

As if the unequal Radii OM and ON drawn from the centre O of any wheel, should sustain the weights A and P, by the cords MA and NP; and the forces of those weights to move the wheel were required. Through the centre O draw the right line KOL, meeting the cords perpendicularly in K and L; and from the centre O, with OL the greater of the distances OK and OL, describe a circle, meeting the cord MA in D: and drawing OD, make AC parallel and DC perpendicular thereto. Now, it being indifferent whether the points K, L, D, of the cords be fixed to the plane of the wheel or not, the weights will have the same effect whether they are suspended from the points K and L, or from D and L. Let the whole force of the weight A be represented by the Line AD, and let it be resolved into the forces AC and CD; of which the force AC, drawing the radius OD directly from the centre, will have no effect to move the wheel: but the other force DC, drawing the radius DO perpendicularly, will have the same effect as if it drew perpendicularly the radius OL equal to OD; that is, it will have the same effect as the weight P, if that weight is to the weight A, as the force DC is to the force DA; that is (because of the similar triangles ADC, DOK,) as OK to OD or OL. Therefore the weights A and P, which are reciprocally as the radii OK

and OL that lye in the same right line, will be equipollent, and so remain in equilibrio: which is the well known property of the Ballance, the Lever, and the Wheel. If either weight is greater than in this ratio, its force to move the wheel will be so much the greater. If the weight p, equal to the weight P, is partly suspended by the cord Np, partly sustained by the oblique plane pG; draw pH, NH, the former perpendicular to the horizon, the latter to the plane pG; and if the force of the weight p tending downwards is represented by the line pH, it may be resolved into the forces pN, HN. If there was any plane perpendicular to the cord pN, cutting the other plane pG in a line parallel to the horizon; and the weight p was supported only by those planes pQ, pG; it would press those planes perpendicularly with the forces pN, HN; to wit, the plane pQ with the force pN, and the plane pG with the force HN. And therefore if the plane pQ was taken away, so that the weight might stretch the cord, because the cord, now sustaining the weight, supplies the place of the plane that was removed, it will be strained by the same force pN which press'd upon the plane before. Therefore the tension of this oblique cord pN will be to that of the other perpendicular cord PN as pN to pH. And therefore if the weight p is to the weight A in a ratio compounded of the reciprocal ratio of the least distances of the cords pN, AM, from the centre of the wheel, and of the direct ratio of pH to pN; the weights will have the same effect towards moving the wheel, and will therefore sustain each other, as any one may find by experiment. But the weight p pressing upon those two oblique planes, may be consider'd as a wedge between the two internal surfaces of a body split by it; and hence the forces of the Wedge and the Mallet may be determin'd; for because the force with which the weight p presses the plane pQ, is to the force with which the same, whether by its own gravity, or by the blow of a mallet, is impelled in the direction of the line pH towards both the planes, as pN to pH; and to the force with which it presses the other plane pG, as pN to NH. And thus the force of the Screw may be deduced from a like resolution of forces; it being no other than a Wedge impelled with the force of a Lever. Therefore the use of this Corollary spreads far and wide, and by that diffusive extent the truth thereof is farther confirmed. For on what has been said depends the whole doctrine of Mechanics variously demonstrated by different authors. For from hence are easily deduced the forces of Machines, which are compounded of Wheels, Pulleys, Leavers, Cords and Weights, ascending directly or obliquely, and other Mechanical Powers; as also the force of the Tendons to move the Bones of Animals.

COROLLARY III. The Quantity of motion, which is collected by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of bodies among themselves.

For Action and its opposite Re-action are equal, by Law 3, and therefore, by Law 2, they produce in the motions equal changes towards opposite parts. Therefore if the motions are directed towards the same parts, whatever is added to the motion of the preceding body will be subducted from the motion of that which follows; so that the sum will be the same as before. If the bodies meet, with contrary motions, there willbe an equal deduction from the motions of both; and therefore the difference of the motions directed towards opposite parts will remain the same. Thus if a sphærical body A with two parts of velocity is triple of a sphærical body B which follows in the same right line with ten parts of velocity; the motion of A will be to that of B, as 6 to 10. Suppose then their motions to be of 6 parts and of 10 parts, and the sum will be 16 parts. Therefore upon the meeting of the bodies, if A acquire 3, 4 or 5 parts of motion, B will lose as many; and therefore after relexion A will proceed with 9, 10 or 11 parts, and B with 7, 6 or 5 parts; the sum remaining always of 16 parts as before. If the body A acquire 9, 10, 11 or 12 parts of motion, and therefore after meeting proceed with 15, 16, 17 or 18 parts; the body B, losing so many parts as A has got, will either proceed with one part, having lost 9; or stop and remain at rest, as having lost its whole progressive motion of 10 parts; or it will go back with one part, having not only lost its whole motion, but (if I may so say) one part more; or it will go back with 2 parts, because a

progressive motion of 12 parts is took off. And so the Sums of the conspiring motions 15 + 1, or 16 + 0, and the Differences of the contrary motions 17 - 1 and 18 - 2 will always be equal to 16 parts, as they were before the meeting and reflexion of the bodies. But, the motions being known with which the bodies proceed after reflexion, the velocity of either will be also known, by taking the velocity after to the velocity before reflexion, as the motion after is to the motion before. As in the last case, where the motion of the body A was of 6 parts before reflexion and of 18 parts after, and the velocity was of 2 parts before reflexion; the velocity thereof after reflexion will be found to be of 6 parts, by saying, as the 6 parts of motion before to 18 parts after, so are 2 parts of velocity before reflexion to 6 parts after. But if the bodies are either not sphærical, or moving in different right lines impinge obliquely one upon the other, and their motions after reflexion are required: in those cases we are first to determine the position of the plane that touches the concurring bodies in the point of concourse; then the motion of each body (by Corol. 2.) is to be resolved into two, one perpendicular to that plane, and the other parallel to it. This done, because the bodies act upon each other in the direction of a line perpendicular to this plane, the parallel motions are to be retained the same after reflexion as before; and to the perpendicular motions we are to assign equal changes towards the contrary pars; in such manner that the sum of the conspiring, and the difference of the contrary motions, may remain the same as before. From such kind of relexions also sometimes arise the circular motions of bodies about their own centres. But these are cases which I don't consider in what follows; and it would be too tedious to demonstrate every particular that relates to this subject.

COROLLARY IV. The common centre of gravity of two or more bodies, does not alter its state of motion or rest by the actions of the bodies among themselves; and therefore the common centre of gravity of all bodies acting upon each other (excluding outward actions and impediments) is either at rest, or moves uniformly in a right line.

For if two points proceed with an uniform motion in right lines, and their distance be divided in a given ratio, the dividing point will be either at rest, or proceed uni-formly in a right line. This is demonstrated hereafter in Lem. 23. and its Corol. When the points are moved in the same plane; and by a like way of arguing, it may be demon-strated when the points are not moved in the same plane. Therefore if any number of bodies move uniformly in right lines, the common centre of gravity of any two of them is either at rest, or proceeds uniformly in a right line; because the line which connects the centres of those two bodies so moving is divided at that common centre in a given ratio. In like manner the common centre of those two and that of a third body will be either at rest or moving uniformly in a right line; because at that centre, the distance between the common centre of the two bodies, and the centre of this last, is divided in a given ratio. In like manner the common centre of these three, and of a fourth body, is either at rest, or moves uniformly in a right line; because the distance between the common centre of the three bodies, and the centre of the fourth is there also divided in a given ratio, and so on *in infinitum*. Therefore in a system of bodies, where there is neither any mutual action among themselves, nor any foreign force impress'd upon them from without, and which consequently move uniformly in right lines, the com-mon centre of gravity of them all is either at rest, or moves uniformly forwards in a right line.

Moreover, in a system of two bodies mutually acting upon each other, since the distances between their centres and the common centre of gravity of both, are reciprocally as the bodies; the relative motions of those bodies, whether of approaching to or of receding from that centre, will be equal among themselves. Therefore since the changes which happen to motions are equal and directed to contrary parts, the common centre of those bodies, by their mutual action between themselves, is neither promoted nor retarded, nor suffers any change as to its state of motion or rest. But in a system of several bodies, because the common centre of gravity of any two acting mutually upon each other suffers no change in

its state by that action; and much less the common centre of gravity of the others with which that action does not intervene; but the distance between those two centres is divided by the common centre of gravity of all the bodies into parts reciprocally proportional to the total sums of those bodies whose centres they are; and therefore while those two centres retain their state of motion or rest, the common centre of all does also retain its state: It is manifest, that the common centre of all never suffers any change in the state of its motion or rest from the actions of any two bodies between themselves. But in such a system all the actions of the bodies among themselves, either happen between two bodies, or are composed of actions interchanged between some two bodies; and therefore they do never produce any alteration in the common centre of all as to its state of motion or rest. Wherefore since that centre when the bodies do not act mutually one upon another, either is at rest or moves uniformly forward in some right line; it will, notwithstanding the mutual actions of the bodies among themselves, always persevere in its state, either of rest, or of proceeding uniformly in a right line, unless it is forc'd out of this state by the action of some power impress'd from without upon the whole system. And therefore the same law takes place in a system, consisting of many bodies, as in one single body, with regard to their persevering in their staet of motion or of rest. For the progressive motion whether of one single body or of a whole system of bodies, is always to be estimated, from the motion of the centre of gravity.

COROLLARY V. The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.

For the differences of the motions tending towards the same parts, and the sums of those that tend towards contrary parts, are at first (by supposition) in both cases the same; and it is from those sums and differences that the collisions and impulses do arise with which the bodies mutually impinge one upon another. Wherefore (by Law 2.) the effects of those collisions will be equal in both cases; and therefore the mutual motions of the bodies among themselves in the one case will remain equal to the mutual motions of the bodies among themselves in the other. A clear proof of which we have from the experiment of a ship: where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line.

COROLLARY VI. If bodies, any how moved among themselves are urged in the direction of parallel lines by equal accelerative forces; they will all continue to move among themselves, after the same manner as if they had been urged by no such forces.

For these forces acting equally (with respect to the quantities of the bodies to be moved) and in the direction of parallel lines, will (by Law 2.) move all the bodies equally (as to velocity) and therefore will never produce any change in the positions or motions of the bodies among themselves.

SCHOLIUM

Hitherto I have laid down such principles as have been receiv'd by Mathematicians, and are confirm'd by abundance of experiments. By the two first Laws and the first two Corollaries, Galileo discover'd that the descent of bodies observ'd the duplicate ratio of the time, and that the motion of projectiles was in the curve of a Parabola; experience agreeing with both, unless so far as these motions are a little retarded by the resistance of the Air. When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities: and in the whole time impresses a whole force and generates a whole velocity, proportional to the time. And the spaces described in proportional times are as the velocities and the times conjunctly; that is, in a duplicate ratio of the times. And when a body is thrown upwards, its uniform gravity impresses forces

and takes off velocities proportional to the times; and the times of ascending to the greatest heights are as the velocities to be taken off, and those heights are as the velocities and the times conjunctly, or in the duplicate ratio of the velocities. And if a body be projected in any direction, the motion arising from its projection is compounded with the motion arising from its gravity. As if the Body A by its motion of projection alone (Fig. 3.) could describe in a given time the right line AB; and with its motion of falling alone could describe in the same time the altitude AC; compleat the paralellogram ABDC, and the body by that compounded will at the end of the time be found in the place D; and the curve line AED, which that body describes, will be a Parabola, to which the right line AB will be a tangent in A; and whose ordinate BD will be as the square of the line AB. On the same laws and corollaries depend those things which have been demonstrated concerning the times of the vibration of Pendulums, and are confirm'd by the daily experiments of Pendulum clocks. By the same together with the third Law Sir Christ. Wren, Dr. Wallis and Mr. Huygens, the greatest Geometers of our times, did severally determine the rules of the Congress and Reflexion of hard bodies, and much about the same time communicated their discoveries to the Royal Society, exactly agreeing among themselves, as to those rules. Dr. Wallis indeed was something more early in the publication; then followed Sir Christopher Wren, and lastly, Mr. Huygens. But Sir Christopher Wren confirmed the truth of the thing before the Royal Society by the experiment of pendulums, which Mr. Mariotte soon after thought fit to explain in a treatise entirely upon that subject. But to bring this experiment to an accurate agreement with the theory, we are to have a due regard as well to the resistance of the air, as to the elastic force of the concurring bodies. Let the sphærical bodies AB, be suspended by the parallel and equal strings, AC, BD, Fig. 4. from the centres C, D. About these centres, with those intervals, describe the semicircles EAF, GBH bisected by the radii CA, DB. Bring the body A to any point R of the arc EAF, and (withdrawing the body B) let it go from thence, and after one oscillation suppose it to return to the point V : then RV will be the retardation arising from the resistance of the air. Of this RV let ST be a fourth part situated in the middle, to wit, so as RS and TV may be equal, and RS may be to ST as 3 to 2: then will ST represent very nearly the retardation during the descent from S to A. Restore the body B to its place: and supposing the body A to be let fall from the point S, the velocity thereof in the place of reflexion A, without sensible error, will be the same as if it had descended *in vacuo* from the point T. Upon which account this velocity may be represented by the chord of the arc TA. For it is a proposition well known to Geometers, that the velocity of a pendulous body in the lowest point is as the chord of the arc which it has described in its descent. After reflexion, suppose the body A comes to the place s, and the body B to the place k. Withdraw the body B, and find the place v, from which if the body A, being let go, should after one oscillation return to the place r, st may be a fourth part of rv, so placed in the middle thereof as to leave rs equal to tv, and let the chord of the arc tA represent the velocity which the body A had in the place A immediately after relexion. For t will be the true and correct place to which the Body A should have ascended, if the resistance of the Air had been taken off. In the same way we are to correct the place k to which the body B ascends, by finding the place l to which it should have ascended *in vacuo*. These things being done we are to take the product (if I may so say) of the body A, by the chord of the arc TA (which represents its velocity) that we may have its motion in the place A immediately before reflexion; and then by the chord of the arc tA, that we may have its motion in the place A immediately after reflexion. And so we are to take the product of the body B by the chord of the arc Bl, that we may have the motion of the same immedately after reflexion. And in like manner, when two bodies are let go together from different places, we are to find the motion of each, as well before as after reflexion; and then we may compare the motions between themselves, and collect the effects of the reflexion. Thus trying the thing with pendulums of ten feet, in unequal as well as equal bodies, and making the bodies to concur after a descent through large spaces, as of 8, 12, or 16 feet, I found always, without an errour of 3 inches, that when the bodies concurr'd together directly, equal changes toward the contrary parts were produced in their motions; and of consequence, that the

action and reaction were always equal. As if the body A imping'd upon the body B at rest with 9 parts of motion, and losing 7, proceeded after reflexion with 2; the body B was carried backwards with those 7 parts. If the bodies concurr'd with contrary motions, A with twelve parts of motion, and B with six, then if A receded with 2, B receded with 8, to wit, with a deduction of 4 parts of motion on each side. For from the motion of A subducting 12 parts, nothing will remain: but subducting 2 parts more, a motion will be generated of 2 parts towards the contrary way; and so, from the motion of the body B of 6 parts, subducting 14 parts, a motion is generated of 8 parts towards the contrary way. But if the bodies were made both to move towards the same way; A, the swifter, with 14 parts of motion, B, the slower, with 5, and after reflexion A went on with 5, B likewise went on with 14 parts; 9 parts being transferr'd from A to B. And so in other cases. By the congress and collision of bodies, the quantity of motion, collected from the sum of the motions directed towards the same way, or from the difference of those that were directed towards contrary ways, was never changed. For the error of an inch or two in measures may be easily ascrib'd to the difficulty of executing every thing with accuracy. It was not easy to let go the two pendulums so exactly together, that the bodies should impinge one upon the other in the lowermost place AB; nor to mark the places s, and k, to which the bodies ascended after congress. Nay, and some errors too might have happen'd from the unequal density of the parts of the pendulous bodies themselves, and from the irregularity of the texture proceeding from other causes. But to prevent an objection that may perhaps be alledged against the rule, for the proof of which this experiment was made, as if this rule did suppose that the bodies were either absolutely hard, or at least perfectly elastic; whereas no such bodies are to be found in nature; I must add that the experiments we have been describing, by no means depending upon that quality of hardness, do succeed as well in soft as in hard bodies. For if the rule is to be tried in bodies not perfectly hard, we are only to diminish the reflexion in such a certain proportion, as the quantity of the elastic force requires. By the theory of Wren and Huygens, bodies absolutely hard return one from another with the same velocity with which they meet. But this may be affirm'd with more certainty of bodies perfectly elastic. In bodies imperfectly elastic the velocity of the return is to be diminish'd together with the elastic force; because that force (except when the parts of bodies are bruised by their congress, or suffer some such extension as happens under the strokes of a hammer,) is (as far as I can perceive) certain and determined, and makes the bodies to return one from the other with a relative velocity, which is in a given ratio to that relative velocity with which they met. This I tried in balls of wool, made up tightly and strongly compress'd. For first, by letting go the pendulous bodies and measuring their reflexion, I determined the quantity of their elastic force; and then, according to this force, estimated the reflexions that ought to happen in other cases of congress. And with this computation other experiments made afterwards did accordingly agree; the balls always receding one from the other with a relative velocity, which was to the relative velocity with which they met, as about 5 to 9. Balls of steel return'd with almost the same velocity: those of cork with a velocity something less: but in balls of glass the proportion was as about 15 to 16. And thus the third law, so far as it regards percussions and reflexions, is prov'd by a theory, exactly agreeing with experience.

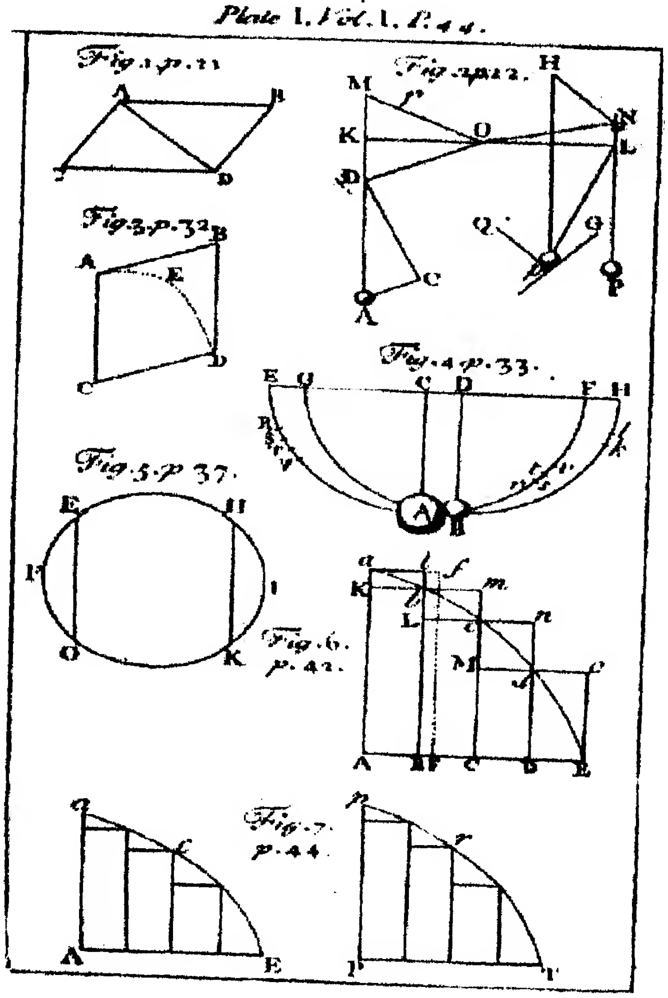
In attractions, I briefly demonstrate the thing after this manner. Suppose an obstacle is interpos'd to hinder the congress of any two bodies A, B, mutually attracting one the other: then if either body as A, is more attracted towards the other body B, than that other body B is towards the first body A, the obstacle will be more strongly urged by the pressure of the body A than by the pressure of the body B; and therefore will not remain in æquilibrio: but the stronger pressure will prevail, and will make the system of the two bodies, together with the obstacle, to move directly towards the parts on which B lies; and in free spaces, to go forwards *in infinitum* with a motion perpetually accelerated. Which is absurd, and contrary to the first law. For by the first law, the system ought to persevere in it's state of rest, or of moving uniformly forward in a right line; and therefore the bodies must equally press the obstacle, and be equally attracted one by the other. I made the experiment on the loadstone and iron. If these plac'd

apart in proper vessels, are made to float by one another in standing water; neither of them will propell the other, but by being equally attracted, they will sustain each others pressure, and rest at last in an equilibrium. So the gravitation betwixt the Earth and its parts, is mutual. Let the Earth FI (Fig. 5.) be cut by any plane EG into two parts EGF and EGI: and their weights one towards the other will be mutually equal. For if by another plane HK, parallel to the former EG, the greater part EGI is cut into two parts EGKH and HKI, whereof HKI is equal to the part EFG first cut off: it is evident that the middle part EGKH will have no propension by its proper weight towards either side, but will hang as it were and rest in an equilibrium betwixt both. But the one extreme part HKI will with its whole weight bear upon and press the middle part toward the other extreme part EGF; and therefore the force, with which EGI, the sum of the parts HKI and EGKH, tends towards the third part EGF, is equal to the weight of the part HKI, that is, to the weight of the third part EGF. And therefore the weights of the two parts EGI and EGF, one towards the other, are equal, as I was to prove. And indeed if those weights were not equal, the whole Earth floating in the non-resisting æther, would give way to the greater weight, and retiring from it, wou'd be carried off *in infinitum*. And as those bodies are equipollent in the congress and reflexion, whose velocities are reciprocally as their innate forces: so in the use of mechanic instruments, those agents are equipollent and mutually sustain each the contrary pressure of the other, whose velocities, estimated according to the determination of the forces, are reciprocally as the forces. So those weights are of equal force to move the arms of a Ballance, which during the play of the ballance are reciprocally as their velocities upwards and downwards:that is, if the ascent or descent is direct, those weights are of equal force, which are reciprocally as the distnaces of the points at which they are suspended from the axe of the ballance; but if they are turned aside by the interposition of oblique planes or other obstacles, and made to ascend or descend obliquely, those bodies will be equipollent, which are reciprocally as the heights of their ascent and descent taken according to the perpendicular; and that on account of the determination of gravity downwards. And in like manner in the Pully, or in a combination of Pullies, the force of a hand drawing the rope directly, that is to the weight, whether ascending directly or obliquely, as the velocity of the perpendicular ascent of the weight to the velocity of the hand that draws the rope, will sustain the weight. In Clocks and such like instruments, made up from a combination of wheels, the contrary forces that promote and impede the motion of the wheels, if they are reciprocally as the velocities of the parts of the wheel on which they are impress'd, will mutually sustain the one the other. The force of the Screw to press a body is to the force of the hand that turnes the handles by which it is moved, as the circular velocity of the handle in that part where it is impelled by the hand, is to the progressive velocity of the Screw towards the press'd body. The forces by which the Wedge presses or drives the two parts of the wood it cleaves, are to the force of the mallet upon the wedge, as the progress of the wedge in the direction of the force impress'd upon it by the mallet, is to the velocity with which the parts of the wood yield to the wedge, in the direction of lines perpendicular to the sides of the wedge. And the like account is to be given of all Machines.

The power and use of Machines consists only in this, that by diminishing the velocity we may augment the force, and the contrary: From whence in all sorts of proper Machines, we have the solution of this problem; *To move a given weight with a given power*, or with a given force to overcome any other given resistance. For if Machines are so contriv'd, that the velocities of the agent and resistant are reciprocally as their forces; the agent will just sustain the resistant: but with a greater disparity of velocity will overcome it. So that if the disparity of velocities is so great, as to overcome all that resistance, which commonly arises either from the attrition of contiguous bodies as they slide by one another, or from the cohesion of continuous bodies that are to be separated, or from the weights of bodies to be raised; the excess of the force remaining, after all those resistances are overcome, will produce an acceleration of motion proportional thereto, as well in the parts of the Machine, as in the resisting body. But to treat of Mechanics is not my present business. I was only willing to shew by those examples, the great extent and

certainty of the third law of motion. For if we estimate the action of the agent from its force and velocity conjunctly; and likewise the re-action of the impediment conjunctly from the velocities of its several parts, and from the forces of resistance arising from the attrition, cohesion, weight, and acceleration of those parts; the action and re-action in the use of all sorts of Machines will be found always equal to one another. And so far as the action is propagated by the intervening instruments, and at last impress'd upon the resisting body, the ultimate determination of the action will be always contrary to the determination of the re-action.

Plate I. Vol. I. P. 44.



After this, Newton goes on to describe the motion of bodies and then to describe gravitational attraction and how planets and comets move.

About the Tamilnadu Science Forum

Tamilnadu Science Forum is a voluntary organization working in education, science communication, health, literacy and women's empowerment. Today the TNSF works in 2000 villages and with about a thousand schools across Tamilnadu. Started by a group of research students and scientists, TNSF grew from a small voluntary network into a mass movement during the literacy campaigns in Tamilnadu. Mobilizing lakhs of learners and volunteers in villages all over Tamilnadu, the TNSF played a critical role in planning and implementing the Mass Literacy Campaigns in over 20,000 villages in 10 districts.

TNSF brings out Thulir and Jantar Mantar - science magazines for children. TNSF has also published a large number of popular science books in Tamil and English. TNSF volunteers work with schools across the state trying to improve the quality of education. Teacher training programmes, joy of learning campaigns, science clubs, children's science festivals and support centers for weaker children and village libraries are just a few of the various educational activities of this Forum.

TNSF also works on women's empowerment and rural development and has organized a large number of poor women in villages into small savings groups. In 800 villages, TNSF also runs a woman and child health programme which has been recognized as one of the best examples of a community health programme. The Science Forum has also been experimenting with eco-friendly, low cost technologies for water, agriculture, sericulture, sanitation and village enterprises in several villages.

About the Association for India's Development

Ten years ago... A far off land. A university town called College Park, near Washington DC. A small bunch of college students from India are discussing...

We must do something for the poor. Let's raise $10 each month from all our friends. We can then support education, health and rural development projects in villages, slums and orphanages in India.

This small beginning created the Association for India's Development. AID raised funds and supported organizations that worked with the poor in villages and slums all over India. AID volunteers who visited village projects, inspired by what they saw, went back to inspire others.

Slowly the organization grew and a number of volunteers from the US came back to India to work in villages and slums - directly and with people's movements like the Tamilnadu Science Forum and the Narmada Bachao Andolan. AID chapters started in Chennai, Bangalore, Bombay and Pune. College students and professionals in these cities joined AID and started raising funds and volunteering their time.

Today AID has over 40 chapters (volunteer groups) spread across the globe. And these volunteers together support over 200 projects reaching out to several lakh poor people all over India. Apart from financially supporting projects, AID volunteers also develop educational materials, work on technology for rural areas, develop experiment kits, answer questions from teachers, coordinate training programmes and volunteer with people's movements in villages.